A figure has **symmetry** if there is a transformation of the figure such that the image and preimage are identical. There are two kinds of symmetry.

<table>
<thead>
<tr>
<th>Line Symmetry</th>
<th>Rotational Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>The figure has a <strong>line of symmetry</strong> that divides the figure into two congruent halves.</td>
<td>When a figure is rotated between $0^\circ$ and $360^\circ$, the resulting figure coincides with the original.</td>
</tr>
<tr>
<td>one line of symmetry</td>
<td>• The smallest angle through which the figure is rotated to coincide with itself is called the <strong>angle of rotational symmetry</strong>.</td>
</tr>
<tr>
<td>two lines of symmetry</td>
<td>• The number of times that you can get an identical figure when repeating the degree of rotation is called the <strong>order</strong> of the rotational symmetry.</td>
</tr>
</tbody>
</table>
| no line symmetry | angle: $180^\circ$  
order: 2 | angle: $120^\circ$  
order: 3 | no rotational symmetry |

Tell whether each figure has line symmetry. If so, draw all lines of symmetry.

1. [Diagram of a triangle]
2. [Diagram of a rectangle]

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

3. [Diagram of a square]
4. [Diagram of a figure with four petals]
Three-dimensional figures can also have symmetry.

<table>
<thead>
<tr>
<th>Symmetry in Three Dimensions</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane Symmetry</td>
<td>A plane can divide a figure into two congruent halves.</td>
<td></td>
</tr>
<tr>
<td>Symmetry About an Axis</td>
<td>There is a line about which a figure can be rotated so that the image and preimage are identical.</td>
<td></td>
</tr>
</tbody>
</table>

A cone has both plane symmetry and symmetry about an axis.

Tell whether each figure has plane symmetry, symmetry about an axis, both, or neither.

5. square pyramid 6. prism

7. triangular pyramid 8. cylinder
12. 180°; 2
13. both
14. plane symmetry
15. both

**Practice B**
1. no
2. yes
3. yes

4. **ANNA BOB OTTO**

5. yes; 180°; 2
6. no
7. yes; 45°; 8
8. 90°; 4

9. neither
10. both
11. plane symmetry

**Practice C**
1. No, a figure cannot have rotational symmetry only at 270° and 360°. Possible answer: If a figure coincides with itself at 270°, then it must coincide with itself at 90°. And if it coincides with itself at 90°, then it must coincide with itself at 180°. The order of rotational symmetry is the number of times a figure coincides with itself as it rotates 360°. The order of rotational symmetry that only occurs at 270° would be \(\frac{360}{270} = \frac{4}{3}\), but a figure cannot coincide with itself one and one-third times during a full rotation.

2. 180°; 2
3. 4. These are concentric circles. Each circle intersects two vertices. Each pair of vertices is on a diameter of a circle, and the pair of vertices switch positions when the polygon is rotated 180° to coincide with itself.

5. 6.

7. 8.

**Reteach**
1. yes; one line of symmetry
2. no
3. yes; 180°; order: 2
4. yes; 90°; order: 4
5. both
6. plane symmetry
7. neither
8. both

**Challenge**
1. TVRG
2. THVRG
3. T
4. TV
5. THG
6. TR
7. Patterns will vary.
8. Answers will vary.
9. For all integers \(n\),

\[
\begin{align*}
  f(x) &= \begin{cases} 
  x - 12n, & \text{where } 12n - 2 \leq x \leq 12n + 2 \\
  2, & \text{where } 12n + 2 \leq x \leq 12n + 4 \\
  -x + 12n + 6, & \text{where } 12n + 4 \leq x \leq 12n + 8 \\
  -2, & \text{where } 12n + 8 \leq x \leq 12n + 10 
  \end{cases}
\end{align*}
\]
Fill in the blanks to complete each definition.

1. The number of times a figure coincides with itself as it rotates through 360° is called the ______________________ of the rotational symmetry.

2. A three-dimensional figure has ______________________ if a plane can divide the figure into two congruent reflected halves.

3. The ______________________ divides a figure into two congruent halves.

4. The angle of rotational symmetry is the ______________________ angle through which a figure can be rotated to coincide with itself.

5. A three-dimensional figure has symmetry about an axis if there is a line about which the figure can be rotated so that the image ______________________ with itself.

Tell whether each figure has line symmetry. If so, draw all lines of symmetry.

6.  
7.  
8.  

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

9.  
10.  
11.  

12. This figure shows the zodiac symbol for Pisces. Draw all lines of symmetry. Give the angle and the order of any rotational symmetry.

Tell whether each figure has plane symmetry, symmetry about an axis, both, or neither.

13.  
14.  
15.  

Challenge

1.  

2.  

3.  

4. When the net is folded, the face with the arrow overlaps the face with the zig zag.

5. The heart is not oriented properly. It must be rotated 180°.

6. The face with the heart has been interchanged with the face with the diamond.

7.  

8.  

9.  

10. Sample answer:

Problem Solving

1.  

2. \(L'(4, -3), M'(-1, 0), N'(4, 1)\)

3. A  

4. G  

5. C  

6. G  

Reading Strategies

1.  

2.  

3.  

4.  

LESSON 12-5

Practice A

1. order  

2. plane symmetry  

3. line of symmetry  

4. smallest  

5. coincides  

6. yes  

7. no  

8. yes  

9. yes; 120°; 3  

10. yes; 180°; 2  

11. no
Tell whether each figure has line symmetry. If so, draw all lines of symmetry.

1. __________________________  2. __________________________

3. __________________________

4. Anna, Bob, and Otto write their names in capital letters. Draw all lines of symmetry for each whole name if possible.

**ANNA**  **BOB**  **OTTO**

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

5. __________________________  6. __________________________  7. __________________________

8. This figure shows the Roman symbol for Earth. Draw all lines of symmetry. Give the angle and order of any rotational symmetry.

__________________________

Tell whether each figure has plane symmetry, symmetry about an axis, both, or neither.

9. __________________________  10. __________________________  11. __________________________
12. 180°; 2
13. both
14. plane symmetry
15. both

Practice B
1. no
2. yes
3. yes

4. ANNA
   BOB
   OTTO

5. yes; 180°; 2
6. no
7. yes; 45°; 8
8. 90°; 4

9. neither
10. both
11. plane symmetry

Practice C
1. No, a figure cannot have rotational symmetry only at 270° and 360°. Possible answer: If a figure coincides with itself at 270°, then it must coincide with itself at 90°. And if it coincides with itself at 90°, then it must coincide with itself at 180°. The order of rotational symmetry is the number of times a figure coincides with itself as it rotates 360°. The order of rotational symmetry that only occurs at 270° would be \( \frac{360}{270} = \frac{4}{3} \), but a figure cannot coincide with itself one and one-third times during a full rotation.

2. 180°; 2

3.

4. These are concentric circles. Each circle intersects two vertices. Each pair of vertices is on a diameter of a circle, and the pair of vertices switch positions when the polygon is rotated 180° to coincide with itself.

5.

6.

7.

8.

Reteach
1. yes; one line of symmetry

2. no
3. yes; 180°; order: 2
4. yes; 90°; order: 4
5. both
6. plane symmetry
7. neither
8. both

Challenge
1. TVRG
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LESSON 12-5

Practice A

Symmetry

Fill in the blanks to complete each definition.

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2. A three-dimensional figure has ______________________ if a plane can divide the figure into two congruent reflected halves.

3. The ______________________ divides a figure into two congruent halves.

4. The angle of rotational symmetry is the ______________________ angle through which a figure can be rotated to coincide with itself.

5. A three-dimensional figure has symmetry about an axis if there is a line about which the figure can be rotated so that the image ______________________ with itself.

Tell whether each figure has line symmetry. If so, draw all lines of symmetry.

6. _______ 7. _______ 8. _______

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

9. ______________________ 10. ______________________ 11. ______________________

12. This figure shows the zodiac symbol for Pisces. Draw all lines of symmetry. Give the angle and the order of any rotational symmetry.

Tell whether each figure has plane symmetry, symmetry about an axis, both, or neither.

13. _______ 14. _______ 15. _______
Challenge

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3. A

4. G

5. C

6. G

Reading Strategies

1. 

2. 

LESSON 12-5

Practice A

1. order

2. plane symmetry

3. line of symmetry

4. smallest

5. coincides

6. yes

7. no

8. yes

9. yes; 120°; 3

10. yes; 180°; 2

11. no
12. 180°; 2
13. both
14. plane symmetry
15. both

Practice B
1. no
2. yes
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4. ANNA BOB OTTO
5. yes; 180°; 2
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