Absolute Value Functions and Graphs

Objective:
I can graph an absolute value function by performing transformations on the parent function $f(x) = |x|$.

| $x$ | $|x|$ | $y$ |
|-----|------|-----|
| -2  |  2   |  2  |
| -1  |  1   |  1  |
|  0  |  0   |  0  |
|  1  |  1   |  1  |
|  2  |  2   |  2  |
Vocabulary

· The function \( f(x) = |x| \) is an absolute value function.

· The highest or lowest point on the graph of an absolute value function is called the vertex.

· An axis of symmetry of the graph of a function is a vertical line that divides the graph into mirror images.

An absolute value graph has one axis of symmetry that passes through the vertex.
Absolute Value Function

- Absolute Value Function
- Vertex \((0, 0)\)
- Axis of Symmetry \(X = 0\)
Building the Absolute Value Function

The absolute value function is defined by $f(x) = |x|$. 

$y = |x|$
This is the absolute value parent function.

Table

| x  | f(x) = |x|   |
|----|-------|-----|
| x  | x > 0 |
| -x | x < 0 |

\[ f(x) = |x| \]
Parent Function

- V-shape
- It is symmetric about the $y$-axis (Axis of Symmetry)
- The vertex is the minimum point on the graph
Translation

A **translation** is a transformation that shifts a graph horizontally or vertically, but doesn’t change the overall shape or orientation.
\[ y = |x| + 7 \]

Vertex: (0, 0)

\[ y = |x - 3| + 0 \]

Vertex: (3, 0)
Absolute value functions and transformations.

\[ y = |x+2| + 3 \]

Vertex \((-2, 3)\)

Points:
- \((0, 5)\)
- \((-2, 1)\)
- \((1, 3)\)
- \((-1, 3)\)
\[ y = |x+3| - 2 \]

(0,0)

(-3,-2)
Stretching and Compression

The graph of \( y = a|x| \) is graph of \( y = |x| \) vertically stretched or compressed depending on the \( |a| \).

| For \( |a| > 1 \) | For \( |a| < 1 \) |
|-----------------|-----------------|
| • The graph is vertically *stretched*, or elongated. | • The graph is vertically *shrunk*, or compressed. |
| • The graph of \( y = a|x| \) is *narrower* than the graph of \( y = |x| \). | • The graph of \( y = a|x| \) is *wider* than the graph of \( y = |x| \). |

The value of \( a \) acts like the slope.
Absolute value functions and transformations.

\[ y = 2|x| \]

Point pairs:
- \((0, 0)\)
- \((1, 1)\)
- \((2, 2)\)
- \((0, 0)\)
- \((-1, 2)\)
- \((-2, -2)\)
\[ y = \frac{1}{2} \left| x + 3 \right| - 4 \]  

\[ \text{(compress)} \]

\[ \text{(0, 0)} \rightarrow \text{(3, 0)} \]

\[ \text{(1, 1)} \rightarrow \text{(2, \frac{1}{2})} \]

\[ \text{(? , ?)} \rightarrow \text{(1, \frac{1}{2})} \]

\[ \frac{1}{2} - 4 \]
Reflection

The graph of $y = a|x|$ is graph of $y = |x|$ reflected across the x-axis.

$$f(x) = |x|$$

$$f(x) = -|x|$$
$y = -a \mid x - h \mid + k$
Multiple Transformations

In general, the graph of an absolute value function of the form $y = a|x - h| + k$ can involve translations, reflections, stretches or compressions.

To graph an absolute value function, start by identifying the vertex
Graphing Absolute Value Functions

Graphing $y = a|x - h| + k$ is straightforward:

1. Plot the vertex $(h, k)$. (note...if $+h$ inside that means $h$ is negative (to the left); if $-h$ inside that means $h$ is positive (to the right)

2. Use the $a$ value as slope to plot more points. Remember you have to do positive and negative slope to get points on both sides of the V

3. Connect the dots in a V-shape.
Example 1

Graph the following function without making a table.

\[ y = |x - 2| + 3 \]

**Vertex**

How does this graph move?

Did this graph stretch or compress?

- \((0, 0) \rightarrow (2, 3)\)
- \((1, 1) \rightarrow (3, 4)\)
- \((2, 2) \rightarrow (4, 5)\)

(slope of 1)
$f(x) = |x|$
Example 2
Graph the following function.
\[ y = \left(\frac{1}{2}\right)|x| \]

Vertex
\((0, 0)\)

How does this graph move?
Did this graph stretch or compress?
• • •
Your turn:

\[ y = \frac{3}{2}|x| \]

Vertex ( , )

Translations

Slope
Your turn:

\[ y = 2|x + 2| - 3 \]

Vertex \((-2, -3)\)

Translations

Slopes
write an abs value function that...

- that stretches by 3
- Reflect over x-axis
- moves 3 left ←<
  and 2 down

\[-3\left|x+3\right|-2\]