Graph each set of ordered pairs. Determine whether the ordered pairs represent a linear function, a quadratic function, or an exponential function.

1. \((-2, 8), (-1, 5), (0, 2), (1, -1)\)

\textbf{SOLUTION:}

\[
\begin{array}{c|c|c|c|c|c|c}
\hline
x & -2 & -1 & 0 & 1 \\
\hline
y & 8 & 5 & 2 & -1 \\
\hline
\end{array}
\]

The points can be connected by a straight line. Thus, the ordered pairs represent a linear function.

2. \((-3, 7), (-2, 3), (-1, 1), (0, 1), (1, 3)\)

\textbf{SOLUTION:}

\[
\begin{array}{c|c|c|c|c|c|c}
\hline
x & -3 & -2 & -1 & 0 & 1 \\
\hline
y & 7 & 3 & 1 & 1 & 3 \\
\hline
\end{array}
\]

A parabola can be drawn with the given points. The curve has a line of symmetry and points are reflected over this line. Thus, the ordered pairs represent a quadratic function.

3. \((-3, 8), (-2, 4), (-1, 2), (0, 1), (1, 0.5)\)

\textbf{SOLUTION:}

\[
\begin{array}{c|c|c|c|c|c|c}
\hline
x & -3 & -2 & -1 & 0 & 1 \\
\hline
y & 8 & 4 & 2 & 1 & 0.5 \\
\hline
\end{array}
\]

A curve can be drawn with the given points. The curve has no line of symmetry and is rapidly decreasing. Thus, the ordered pairs represent an exponential function.
4. (0, 2), (1, 2.5), (2, 3), (3, 3.5)

**SOLUTION:**

![Graph](image)

The points can be connected by a straight line. Thus, the ordered pairs represent a linear function.

**Look for a pattern in each table of values to determine which kind of model best describes the data.**

5.

**SOLUTION:**

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>8</td>
<td>17</td>
<td>32</td>
<td>53</td>
</tr>
</tbody>
</table>

First differences:

<table>
<thead>
<tr>
<th>5</th>
<th>8</th>
<th>17</th>
<th>32</th>
<th>53</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>9</td>
<td>15</td>
<td>21</td>
<td></td>
</tr>
</tbody>
</table>

Second differences:

<table>
<thead>
<tr>
<th>3</th>
<th>6</th>
<th>6</th>
</tr>
</thead>
</table>

Since the second differences are equal, the table of values represents a quadratic function.

6.

**SOLUTION:**

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-6.75</td>
<td>-7.5</td>
<td>-8.25</td>
<td>-9</td>
</tr>
</tbody>
</table>

First differences:

<table>
<thead>
<tr>
<th>-6.75</th>
<th>-7.5</th>
<th>-8.25</th>
<th>-9</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.75</td>
<td>-0.75</td>
<td>-0.75</td>
<td>----</td>
</tr>
</tbody>
</table>

Since the first differences are equal, the table of values represents a linear function.
9-6 Analyzing Functions with Successive Differences

7. **SOLUTION:**

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>6</td>
<td>12</td>
<td>24</td>
<td>48</td>
</tr>
</tbody>
</table>

First differences:

\[
\begin{array}{ccccccc}
3 & 6 & 12 & 24 & 48 \\
3 & 6 & 12 & 24 & \\
3 & 6 & 12 & \\
\end{array}
\]

Second differences:

\[
\begin{array}{ccccccc}
3 & 6 & 12 & 24 & 48 \\
3 & 6 & 12 & \\
3 & 6 & \\
\end{array}
\]

Ratios of successive y-values:

\[
\begin{array}{ccccccc}
3 & 6 & 12 & 24 & 48 \\
\frac{6}{3} = 2 & \frac{12}{6} = 2 & \frac{24}{12} = 2 & \frac{48}{24} = 2 \\
\end{array}
\]

Since the ratios of successive y-values are equal, the table of values represents an exponential function.

8. **SOLUTION:**

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-1.5</td>
<td>0.5</td>
<td>2.5</td>
<td>6</td>
<td>10.5</td>
</tr>
</tbody>
</table>

First differences:

\[
\begin{array}{ccccccc}
-1.5 & 0 & 2.5 & 6 & 10.5 \\
1.5 & 2.5 & 3.5 & \\
\end{array}
\]

Second differences:

\[
\begin{array}{ccccccc}
1.5 & 2.5 & 3.5 & 4.5 \\
1 & 1 & 1 & \\
\end{array}
\]

Since the second differences are equal, the table of values represents a quadratic function.
9-6 Analyzing Functions with Successive Differences

Determine which kind of model best describes the data. Then write an equation for the function that models the data.

9.

SOLUTION:

First differences:

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td>27</td>
<td>81</td>
</tr>
</tbody>
</table>

Second differences:

<table>
<thead>
<tr>
<th>1</th>
<th>3</th>
<th>9</th>
<th>27</th>
<th>81</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
<td>18</td>
<td>54</td>
<td></td>
</tr>
</tbody>
</table>

Ratios of successive y-values:

1 3 9 27 81

Since the ratios of successive y-values are equal, the table of values represents an exponential function of the form \( y = ab^x \). The constant ratio is 3, so \( b = 3 \). To find \( a \), use an ordered pair from the table, such as (0, 3).

\[
\begin{align*}
y &= ab^x \\
3 &= a(3)^0 \\
3 &= a(1) \\
3 &= a
\end{align*}
\]

An equation that models the data is \( y = 3 \cdot 3^x \).
Graph each set of ordered pairs. Determine whether the ordered pairs represent a linear function, a quadratic function, ...

10. SOLUTION:

First differences:

\[
\begin{array}{cccc}
125 & 80 & 45 & 20 & 5 \\
\end{array}
\]

Second differences:

\[
\begin{array}{cccc}
-45 & -35 & -25 & -15 \\
\end{array}
\]

Since the second differences are equal, the table of values represents a quadratic function of the form \( y = ax^2 \). To find \( a \), use an ordered pair from the table, such as \((-1, 5)\).

\[
y = ax^2
\]

\[
5 = a(-1)^2
\]

\[
5 = a
\]

An equation that models the data is \( y = 5x^2 \).
9-6 Analyzing Functions with Successive Differences

11.

**SOLUTION:**

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>2.5</td>
<td>3</td>
</tr>
</tbody>
</table>

First differences:

\[
\begin{array}{cccc}
1 & 0.5 & 0.5 & 0.5 \\
1.5 & 2 & 2.5 & 3 \\
0.5 & 0.5 & 0.5 & \\
\end{array}
\]

Since the first differences are equal, the table of values represents a linear function of the form \( y = mx + b \). The first difference represents the slope. To find \( b \), use an ordered pair from the table, such as \((1, 3)\).

\[
y = mx + b
\]

\[
3 = \frac{1}{2}(1) + b
\]

\[
3 = \frac{1}{2} + b
\]

\[
\frac{5}{2} = b
\]

An equation that models the data is \( y = \frac{1}{2}x + \frac{5}{2} \).

12.

**SOLUTION:**

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-1.25</td>
<td>-1</td>
<td>-0.75</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

First differences:

\[
\begin{array}{cccc}
-1.25 & -1 & -0.75 & -0.5 \\
-1 & -0.75 & -0.5 & \\
-1 & -0.75 & -0.5 & \\
\end{array}
\]

Since the first differences are equal, the table of values represents a linear function of the form \( y = mx + b \). The first difference represents the slope. To find \( b \), use an ordered pair from the table, such as \((0, -1)\).

\[
y = mx + b
\]

\[
-1 = \frac{1}{4}(0) + b
\]

\[
-1 = b
\]

An equation that models the data is \( y = \frac{1}{4}x - 1 \).
9-6 Analyzing Functions with Successive Differences

13. **PLANTS** The table shows the height of a plant for four consecutive weeks. Determine which kind of function best models the height. Then write a function that models the data.

<table>
<thead>
<tr>
<th>Week</th>
<th>Height (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>3.5</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

**SOLUTION:**

First differences: 0.5   0.5   0.5   0.5

Since the first differences are equal, the table of values represents a linear function of the form $y = mx + b$. The first difference represents the slope. To find $b$, use an ordered pair from the table, such as $(0, 3)$.

$$y = mx + b$$

$$3 = 0.5(0) + b$$

$$b = 3$$

An equation that models the data is $y = 0.5x + 3$.

**Graph each set of ordered pairs. Determine whether the ordered pairs represent a linear function, a quadratic function, or an exponential function.**

14. $(-1, 1), (0, -2), (1, -3), (2, -2), (3, 1)$

**SOLUTION:**

A parabola can be drawn with the given points. The curve has a line of symmetry and points are reflected over this line. Thus, the ordered pairs represent a quadratic function.

15. $(1, 2.75), (2, 2.5), (3, 2.25), (4, 2)$

**SOLUTION:**

The points can be connected by a straight line. Thus, the ordered pairs represent a linear function.
9-6 Analyzing Functions with Successive Differences

16. (-3, 0.25), (-2, 0.5), (-1, 1), (0, 2)

**SOLUTION:**

A curve can be drawn with the given points. The curve has no line of symmetry and is rapidly increasing. Thus, the ordered pairs represent an exponential function.

17. (-3, -11), (-2, -5), (-1, -3), (0, -5)

**SOLUTION:**

A parabola can be drawn with the given points. The curve has a line of symmetry and points are reflected over this line. Thus, the ordered pairs represent a quadratic function.

18. (-2, 6), (-1, 1), (0, -4), (1, -9)

**SOLUTION:**

The points can be connected by a straight line. Thus, the ordered pairs represent a linear function.
9-6 Analyzing Functions with Successive Differences

19. (−1, 8), (0, 2), (1, 0.5), (2, 0.125)

SOLUTION:

A curve can be drawn with the given points. The curve has no line of symmetry and is rapidly decreasing. Thus, the ordered pairs represent an exponential function.

Look for a pattern in each table of values to determine which kind of model best describes the data. Then write an equation for the function that models the data.

20.

SOLUTION:

First differences:

\[ -8.8 \quad -8.6 \quad -8.4 \quad -8.2 \]

\[
\begin{array}{c}
0.2 \\
0.2 \\
0.2
\end{array}
\]

Since the first differences are equal, the table of values represents a linear function of the form \( y = mx + b \). The first difference represents the slope. To find \( b \), use an ordered pair from the table, such as \((0, -8.2)\).

\[
y = mx + b
\]
\[
-8.2 = 0.2(0) + b
\]
\[
-8.2 = b
\]

An equation that models the data is \( y = 0.2x - 8.2 \).
9-6 Analyzing Functions with Successive Differences

21. SOLUTION:

\[
\begin{array}{c|c|c|c|c|c|c|c}
 x & -2 & -1 & 0 & 1 & 2 \\
 y & 10 & 2.5 & 0 & 2.5 & 10 \\
\end{array}
\]

First differences:
\[
\begin{array}{c|c|c|c|c|c|c|c}
 & 10 & 2.5 & 0 & 2.5 & 10 \\
\end{array}
\]

Second differences:
\[
\begin{array}{c|c|c|c|c|c|c|c}
 & -7.5 & -2.5 & 2.5 & 7.5 \\
\end{array}
\]

Since the second differences are equal, the table of values represents a quadratic function of the form \( y = ax^2 \). To find \( a \), use an ordered pair from the table, such as (2, 10).

\[
y = ax^2
\]

\[
10 = a(2)^2
\]

\[
10 = a(4)
\]

\[
2.5 = a
\]

An equation that models the data is \( y = 2.5x^2 \).
22.  

**SOLUTION:**

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0.75</td>
<td>3</td>
<td>12</td>
<td>48</td>
<td>192</td>
</tr>
</tbody>
</table>

First differences:

<table>
<thead>
<tr>
<th>0.75</th>
<th>3</th>
<th>12</th>
<th>48</th>
<th>192</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.25</td>
<td>9</td>
<td>36</td>
<td>144</td>
<td></td>
</tr>
</tbody>
</table>

Second differences:

<table>
<thead>
<tr>
<th>2.25</th>
<th>9</th>
<th>36</th>
<th>144</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.75</td>
<td>27</td>
<td>108</td>
<td></td>
</tr>
</tbody>
</table>

Ratios of successive y-values:

\[
\frac{3}{0.75} = 4, \quad \frac{12}{3} = 4, \quad \frac{48}{12} = 4, \quad \frac{192}{48} = 4
\]

Since the ratios of successive y-values are equal, the table of values represents an exponential function of the form \( y = ab^x \). The constant ratio is 4, so \( b = 4 \). To find \( a \), use an ordered pair from the table, such as \((0, 3)\).

\[
y = ab^x
\]

\[
3 = a(4)^0
\]

\[
3 = a(1)
\]

\[
3 = a
\]

An equation that models the data is \( y = 3 \cdot 4^x \).
23.

**SOLUTION:**

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0.008</td>
<td>0.04</td>
<td>0.2</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

First differences:

0.008 0.04 0.2 1 5

Second differences:

0.032 0.16 0.8 4

Ratios of successive y-values:

0.008 0.04 0.2 1 5

\[
\frac{0.04}{0.008} = 5 \quad \frac{0.2}{0.04} = 5 \quad \frac{1}{0.2} = 5 \quad \frac{5}{1} = 5
\]

Since the ratios of successive y-values are equal, the table of values represents an exponential function of the form \( y = ab^x \). The constant ratio is 5, so \( b = 5 \). To find \( a \), use an ordered pair from the table, such as (2, 5).

\[
y = ab^x
\]

\[
5 = a(5)^2
\]

\[
5 = a(25)
\]

\[
a = 0.2
\]

An equation that models the data is \( y = 0.2 \cdot 5^x \).
24.

**SOLUTION:**

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0</td>
<td>4.2</td>
<td>16.8</td>
<td>37.8</td>
<td>67.2</td>
</tr>
</tbody>
</table>

First differences:

\[
\begin{array}{cccc}
0 & 4.2 & 16.8 & 37.8 & 67.2 \\
4.2 & 12.6 & 21 & 29.4 \\
& 8.4 & 8.4 & 8.4 \\
\end{array}
\]

Second differences:

\[
\begin{array}{cccc}
4.2 & 12.6 & 21 & 29.4 \\
& 8.4 & 8.4 & 8.4 \\
\end{array}
\]

Since the second differences are equal, the table of values represents a quadratic function of the form $y = ax^2$. To find $a$, use an ordered pair from the table, such as $(1, 4.2)$.

\[
y = ax^2 \\
4.2 = a(1)^2 \\
4.2 = a(1) \\
4.2 = a \\
\]

An equation that models the data is $y = 4.2x^2$.

25.

**SOLUTION:**

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>14.75</td>
<td>9.75</td>
<td>4.75</td>
<td>-0.25</td>
<td>-5.25</td>
</tr>
</tbody>
</table>

First differences:

\[
\begin{array}{cccc}
14.75 & 9.75 & 4.75 & -0.25 & -5.25 \\
& -5 & -5 & -5 \\
\end{array}
\]

Since the first differences are equal, the table of values represents a linear function of the form $y = mx + b$. The first difference represents the slope. To find $b$, use an ordered pair from the table, such as $(0, -0.25)$.

\[
y = mx + b \\
-0.25 = -5(0) + b \\
-0.25 = b \\
\]

An equation that models the data is $y = -5x - 0.25$. 

---

9-6 Analyzing Functions with Successive Differences

---

A company launched a new Web site. They tracked the number of visitors to its Web site over a period of time. The data is shown in the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0</td>
<td>4.2</td>
<td>16.8</td>
<td>37.8</td>
<td>67.2</td>
</tr>
</tbody>
</table>

Determine whether the data represents a linear function, a quadratic function, or an exponential function.

**SOLUTION:**

- **Linear Function:**
  - First differences:
    - First row: 0, 4.2, 16.8, 37.8, 67.2
    - Second row: 4.2, 12.6, 21, 29.4
    - Third row: 8.4, 8.4, 8.4
  - Since the second differences are equal, the table of values represents a quadratic function of the form $y = ax^2$. To find $a$, use an ordered pair from the table, such as $(1, 4.2)$.
    
    \[
y = ax^2 \\
4.2 = a(1)^2 \\
4.2 = a \\
\]

  - An equation that models the data is $y = 4.2x^2$.

- **Quadratic Function:**
  - Since the second differences are equal, the table of values represents a quadratic function. An equation that models the data is $y = 4.2x^2$.

- **Exponential Function:**
  - Ratios of successive values:
    - First row: 0, 4.2, 16.8, 37.8, 67.2
    - Second row: 4.2, 12.6, 21, 29.4
    - Third row: 8.4, 8.4, 8.4
  - Since the ratios of successive values are equal, the table of values represents an exponential function.

An equation that models the data is $y = 4.2x^2$.
26. **WEB SITES** A company launched a new Web site. They tracked the number of visitors to its Web site over a period of 4 days. Determine which kind of model best represents the number of visitors to the website with respect to time. Then write a function that models the data.

<table>
<thead>
<tr>
<th>Day</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visitors (in thousands)</td>
<td>0</td>
<td>0.9</td>
<td>3.6</td>
<td>8.1</td>
<td>14.4</td>
</tr>
</tbody>
</table>

**SOLUTION:**

<table>
<thead>
<tr>
<th>Day</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visitors (in thousands)</td>
<td>0</td>
<td>0.9</td>
<td>3.6</td>
<td>8.1</td>
<td>14.4</td>
</tr>
</tbody>
</table>

First differences:

\[
\begin{array}{cccc}
0 & 0.9 & 3.6 & 8.1 & 14.4 \\
0.9 & 2.7 & 4.5 & 6.3 & \\
\end{array}
\]

Second differences:

\[
\begin{array}{cccccc}
0.9 & 2.7 & 4.5 & 6.3 & \\
1.8 & 1.8 & 1.8 & \\
\end{array}
\]

Since the second differences are equal, the table of values represents a quadratic function of the form \( y = ax^2 \). To find \( a \), use an ordered pair from the table, such as (1, 0.9).

\[
y = ax^2 \\
0.9 = a(1)^2 \\
0.9 = a(1) \\
0.9 = a
\]

An equation that models the data is \( y = 0.9x^2 \).
27. **LONG DISTANCE** The cost of a long-distance telephone call depends on the length of the call. The table shows the cost for up to 6 minutes.

<table>
<thead>
<tr>
<th>Length of call (min)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>0.12</td>
<td>0.24</td>
<td>0.36</td>
<td>0.48</td>
<td>0.60</td>
<td>0.72</td>
</tr>
</tbody>
</table>

a. Graph the data and determine which kind of function best models the data.
b. Write an equation for the function that models the data.
c. Use your equation to determine how much a 10-minute call would cost.

**SOLUTION:**

a. The points can be connected by a straight line. Thus, the ordered pairs represent a linear function.

b. The point (0, 0) is appears to be on the graph. Thus, the y-intercept is 0.

To find \( m \), use two ordered pair from the table, such as \((1, 0.12)\) and \((2, 0.24)\).

\[
m = \frac{0.24 - 0.12}{2 - 1} = \frac{0.12}{1} = 0.12
\]

An equation that models the data is \( y = 0.12x \).

c. Let \( x = 10 \).

\[
y = mx + b
0.12 = 0.12(1) + b
0.12 = 0.12 + b
0 = b
\]

So, a 10-minute long-distance phone call would cost $1.20.
28. **DEPRECIATION** The value of a car depreciates over time. The table shows the value of a car over a period of time.

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value ($)</td>
<td>18,500</td>
<td>15,910</td>
<td>13,682.60</td>
<td>11,767.04</td>
<td>10,119.65</td>
</tr>
</tbody>
</table>

a. Determine which kind of function best models the data.
b. Write an equation for the function that models the data.
c. Use your equation to determine how much the car is worth after 7 years.

**SOLUTION:**

First differences:

\[18,500 - 15,910 - 13,682.60 - 11,767.04 - 10,119.65\]

Second differences:

\[2590 - 2227.4 - 1915.56 - 1647.39\]

Ratios of successive \(v\)-values:

\[\frac{15910}{18500} = 0.86, \quad \frac{13682.6}{15910} = 0.86, \quad \frac{11767.04}{13682.6} = 0.86, \quad \frac{10119.65}{11767.04} = 0.86\]

Since the ratios of successive \(v\)-values are equal, the table of values represents an exponential function of the form \(v = ab^t\), where \(v\) is the value of the car after \(t\) years.

b. The constant ratio is 0.86, so \(b = 0.86\). \(a\) is the initial value of the car, or $18,500. So, the function that best models the data is \(v = 18,500(0.86)^t\).

c.

\[v = 18,500(0.86)^t\]
\[v = 18,500(0.86)^7\]
\[v = 18,500(0.3479278222)\]
\[v \approx 6436.66\]

So, the car is worth about $6436.66 after 7 years.
29. **BACTERIA** A scientist estimates that a bacteria culture with an initial population of 12 will triple every hour.
   
a. Make a table to show the bacteria population for the first 4 hours.
   
b. Which kind of model best represents the data?
   
c. Write a function that models the data.
   
d. How many bacteria will there be after 8 hours?

**SOLUTION:**

a. 

<table>
<thead>
<tr>
<th>Time (hour)</th>
<th>Amount of Bacteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>36</td>
</tr>
<tr>
<td>2</td>
<td>108</td>
</tr>
<tr>
<td>3</td>
<td>324</td>
</tr>
<tr>
<td>4</td>
<td>972</td>
</tr>
</tbody>
</table>

b. Because the successive $b$-values triple, there is a constant ratio of 3 and the best model is an exponential function of the form $b = ac^t$, where $b$ is the amount of bacteria and $t$ is the time in hours.

c. The constant ratio is 3, so $c = 3$. To find $a$, use an ordered pair from the table, such as $(0, 12)$.

$$b = ac^t$$

$$12 = a(3)^0$$

$$12 = a$$

So, a function that models the data is $b = 12 \cdot 3^t$.

d. To find how many bacteria there will be after 8 hours, let $t = 8$.

$$b = 12 \cdot 3^8$$

$$b = 12 \cdot 6561$$

$$b = 78,732$$

So, there will be 78,732 bacteria after 8 hours.

30. **PRINTING** A printing company charges the fees shown to print flyers. Write a function that models the total cost of the flyers, and determine how much 30 flyers would cost.

**SOLUTION:**

Because the total cost of the flyers goes up $0.15 with each additional flyer, a linear function best models the situation. Let $C$ be the total cost of the flyers and $t$ be the number of flyers printed. The cost of each flyer, $0.15$, represents the slope, and $25$ represents the $C$-intercept, the cost of 0 flyers. So, a function that models the total cost of the flyers is $C = 0.15t + 25$.

To find the total cost of 30 flyers, let $t = 30$.

$$C = 0.15t + 25$$

$$C = 0.15(30) + 25$$

$$C = 4.5 + 25$$

$$C = 29.5$$

So, Christine will be charged $29.50 if she orders 30 flyers.
31. **CHALLENGE** Write a function that has constant second differences, first differences that are not constant, a y-intercept of −5, and contains the point (2, 3).

**SOLUTION:**
Sample answer: If the second differences are constant but the first differences are not constant, then the function is a quadratic. Since the y-intercept is not 0, the function will have the form \( y = ax^2 + c \), where \( c \) is the y-intercept. Use the point (2, 3) to determine the value of \( a \) for the function.

\[
y = ax^2 + c \quad \text{Form of quadratic function}
\]

\[
y = ax^2 - 5 \quad c = -5
\]

\[
3 = a(2)^2 - 5 \quad (x, y) = (2, 3)
\]

\[
3 = 4a - 5 \quad \text{Multiply.}
\]

\[
8 = 4a \quad \text{Add 5 to each side.}
\]

\[
2 = a \quad \text{Divide each side by 4.}
\]

So, a function has constant second differences, first differences that are not constant, a y-intercept of -5, and contains the point (2, 3) could be \( y = 2x^2 - 5 \).

32. **CCSS ARGUMENTS** What type of function will have constant third differences but not constant second differences? Explain.

**SOLUTION:**
Sample answer: Linear functions have a constant first difference and quadratic functions have a constant second difference, so cubic equations would have a constant third difference.

Consider the following data for \( y = x^3 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-1)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>27</td>
</tr>
</tbody>
</table>

First differences:

\[ -1 \ 0 \ 1 \ 8 \ 27 \]

Second differences:

\[ 1 \ 1 \ 7 \ 19 \]

Third differences:

\[ 0 \ 6 \ 12 \ 6 \]

Therefore the data represents a cubic function.

33. **OPEN ENDED** Write a linear function that has a constant first difference of 4.

**SOLUTION:**
The constant first difference represents the slope, so any linear function with a slope of 4 will work. Sample answer: \( y = 4x + 1 \)
34. **PROOF** Write a paragraph proof to show that linear functions grow by equal differences over equal intervals, and exponential functions grow by equal factors over equal intervals. (Hint: Let \( y = ax \) represent a linear function and let \( y = a^x \) represent an exponential function.)

**SOLUTION:**
Consider a generic linear function \( y = ax \) over different intervals \( 0 < x < 1 \) and \( 1 < x < 2 \). For the first interval, the difference in \( y \) is:
\[
a(1) - [a(0)] = a
\]
For the second interval, the difference in \( y \) is:
\[
a(2) - [a(1)] = 2a - a = a
\]
So we see that for any interval of length 1, the difference in \( y \) is \( a \).

Consider an exponential function of \( y = a^x \) over the same intervals. For the first interval, the factor by which the function grows is:
\[
\frac{a^1}{a^0} = a
\]
And the factor of growth over the second interval is:
\[
\frac{a^2}{a^1} = a
\]
So for any interval of length 1, we see that the factor of growth for the exponential function is \( a \).
35. **WRITING IN MATH** How can you determine whether a given set of data should be modeled by a linear function, a quadratic function, or an exponential function?

**SOLUTION:**

The data can be graphed to determine a trend, or the differences or ratios between successive terms can be compared. For example:

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-8</td>
<td>-3</td>
<td>2</td>
<td>7</td>
<td>12</td>
</tr>
</tbody>
</table>

First differences: \(-8 \Rightarrow -3 \Rightarrow 2 \Rightarrow 7 \Rightarrow 12\)

Since the first differences are all equal, the table of values represents a linear function.

Determine which model fits the data.

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

First differences: \(8 \Rightarrow 4 \Rightarrow 2 \Rightarrow 1 \Rightarrow 0.5\)

The first differences are not all equal. So, the table of values does not represent a linear function. Find the second differences and compare.

Second differences:

The second differences are not all equal. So, the table of values does not represent a quadratic function. Find the ratios of the y-values and compare.

Ratios: \(\frac{8}{8} = 1 \Rightarrow \frac{4}{4} = 1 \Rightarrow \frac{2}{2} = 1 \Rightarrow \frac{1}{1} = 1 \Rightarrow \frac{0.5}{0.5} = 1\)

The ratios of successive y-values are equal. Therefore, the table of values can be modeled by an exponential function.
36. **SHORT RESPONSE** Write an equation that models the data provided in the table.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>6</td>
<td>12</td>
<td>24</td>
<td>48</td>
</tr>
</tbody>
</table>

**SOLUTION:**

First difference:

<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>6</th>
<th>12</th>
<th>24</th>
<th>48</th>
</tr>
</thead>
</table>

Second difference:

<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>6</th>
<th>12</th>
</tr>
</thead>
</table>

Ratios of successive y-values:

<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>6</th>
<th>12</th>
<th>24</th>
<th>48</th>
</tr>
</thead>
</table>

Since the ratios of successive y-values are equal, the table of values represents an exponential function of the form $y = ab^x$. The constant ratio is 2, so $b = 2$. To find $a$, use an ordered pair from the table, such as (0, 3).

$y = ab^x$

$3 = a(2)^0$

$3 = a(1)$

$3 = a$

An equation that models the data is $y = 3 \cdot 2^x$. 
37. What is the equation of the line below?

[Graph showing a straight line]

A  \( y = \frac{2}{5}x + 2 \)

B  \( y = \frac{2}{5}x - 2 \)

C  \( y = \frac{5}{2}x + 2 \)

D  \( y = \frac{5}{2}x - 2 \)

**SOLUTION:**
The \( y \)-intercept of the line is 2, so choices B and D can be eliminated. Using the \( x \)- and \( y \)-intercepts from the graph, \((-5, 0)\) and \((0, 2)\), determine the slope.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{0 - (-5)} = \frac{2}{5}
\]

Because the slope is \( \frac{2}{5} \) and the \( y \)-intercept is 2, the equation of the line is \( y = \frac{2}{5}x + 2 \), or choice A.

38. The point \((r, -4)\) lies on a line with an equation of \(2x + 3y = -8\). Find the value of \(r\).

F  10

G  0

H  2

J  8

**SOLUTION:**
Let \( x = r \) and let \( y = -4 \).

\[2x + 3y = -8\]

\[2r + 3(-4) = -8\]

\[2r - 12 = -8\]

\[2r = 4\]

\[r = 2\]

So, the correct choice is H.
39. **GEOMETRY** The rectangle has an area of 220 square feet. Find the length \( \ell \).

\[ \ell \times (\ell + 12) = 220 \]

\[ \ell^2 + 12\ell = 220 \]

\[ 0 = \ell^2 + 12\ell - 220 \]

\[ 0 = (\ell + 22)(\ell - 10) \]

\[ \ell = -22 \text{ or } \ell = 10 \]

Because the length of a rectangle must be positive, the length of the rectangle is 10 feet. So, the correct choice is B.

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

40. \( 6x^2 - 3x - 30 = 0 \)

**SOLUTION:**

For this equation, \( a = 6 \), \( b = -3 \), and \( c = -30 \).

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(6)(-30)}}{2(6)} \]

\[ x = \frac{3 \pm \sqrt{9 + 720}}{12} \]

\[ = \frac{3 \pm \sqrt{729}}{12} \]

\[ = \frac{3 \pm 27}{12} \]

\[ = \frac{3 + 27}{12} \text{ or } \frac{3 - 27}{12} \]

\[ x = 2.5 \text{ or } x = -2 \]

The solutions are 2.5 and -2.
9-6 Analyzing Functions with Successive Differences

41. \(4x^2 + 18x = 10\)

**SOLUTION:**

Write the equation in standard form.

\[4x^2 + 18x = 10\]

\[4x^2 + 18x - 10 = 0\]

For this equation, \(a = 4, b = 18,\) and \(c = -10.\)

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[= \frac{-18 \pm \sqrt{(18)^2 - 4(4)(-10)}}{2(4)}\]

\[= \frac{-18 \pm \sqrt{324 + 160}}{8}\]

\[= \frac{-18 \pm \sqrt{484}}{8}\]

\[= \frac{-18 \pm 22}{8}\]

\[x = \frac{-18 + 22}{8} \text{ or } x = \frac{-18 - 22}{8}\]

\[= 0.5 \text{ or } x = -5\]

The solutions are 0.5 and -5.

42. \(2x^2 + 6x = 7\)

**SOLUTION:**

Write the equation in standard form.

\[2x^2 + 6x = 7\]

\[2x^2 + 6x - 7 = 0\]

For this equation, \(a = 2, b = 6,\) and \(c = -7.\)

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[= \frac{-6 \pm \sqrt{(6)^2 - 4(2)(-7)}}{2(2)}\]

\[= \frac{-6 \pm \sqrt{36 + 56}}{4}\]

\[= \frac{-6 \pm \sqrt{92}}{4}\]

\[= \frac{-6 \pm 9.59}{4}\]

\[x = \frac{-6 + 9.59}{4} \text{ or } x = \frac{-6 - 9.59}{4}\]

\[\approx 0.9 \text{ or } x = -3.9\]

The solutions are 0.9 and -3.9.
Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

43. \( x^2 = 25 \)

**SOLUTION:**

\[
x^2 = 25
\]

\[
\sqrt{x^2} = \sqrt{25}
\]

\[
x = \pm 5
\]

44. \( x^2 + 6x + 9 = 16 \)

**SOLUTION:**

\[
x^2 + 6x + 9 = 16
\]

\[
(x + 3)^2 = 16
\]

\[
\sqrt{(x + 3)^2} = \sqrt{16}
\]

\[
x + 3 = \pm 4
\]

\[
x = -3 \pm 4
\]

\[
x = 1 \quad \text{or} \quad x = -7
\]

45. \( x^2 - 14x + 49 = 15 \)

**SOLUTION:**

\[
x^2 - 14x + 49 = 15
\]

\[
(x - 7)^2 = 15
\]

\[
\sqrt{(x - 7)^2} = \sqrt{15}
\]

\[
x - 7 = \pm 3.87
\]

\[
x = 7 \pm 3.87
\]

\[
x = 10.9 \quad \text{or} \quad x = 3.1
\]

46. **INVESTMENTS** Joey’s investment of $2500 has been losing its value at a rate of 1.5% each year. What will his investment be worth in 5 years?

**SOLUTION:**

Using the equation exponential decay, let \( a = 2500 \), \( r = 1.5\% = 0.015 \), and \( t = 5 \).

\[
y = a(1 - r)^t
\]

\[
y = 2500(1 - 0.015)^5
\]

\[
y = 2500(0.985)^5
\]

\[
y \approx 2318.04
\]

So, Joey’s investment will be worth about $2318.04 in 5 years.
9-6 Analyzing Functions with Successive Differences

Write an equation for the \( n \)th term of each geometric sequence, and find the seventh term of each sequence.

47. \( 1, 2, 4, 8, \ldots \)

**SOLUTION:**
The first term of the sequence is 1. So, \( a_1 = 1 \).
Calculate the common ratio.

\[
\begin{array}{cccc}
1 & 2 & 4 & 8 \\
\frac{2}{1} & = & 2 & \frac{4}{2} = 2 & \frac{8}{4} = 2
\end{array}
\]

The common ratio is 2. So, \( r = 2 \).
\[
a_n = a_1 r^{n-1}
\]
\[
a_n = 1(2)^{n-1}
\]
To find the seventh term of the sequence, let \( n = 7 \).
\[
a_n = 1(2)^{7-1}
\]
\[
= 2^6
\]
\[
= 64
\]
So, the seventh term is 64.

48. \( -20, -10, -5, \ldots \)

**SOLUTION:**
The first term of the sequence is \(-20\). So, \( a_1 = -20 \).
Calculate the common ratio.

\[
\begin{array}{cccc}
-20 & -10 & -5 \\
\frac{-10}{-20} = 0.5 & \frac{-5}{-10} = 0.5
\end{array}
\]

The common ratio is 0.5. So, \( r = 0.5 \).
\[
a_n = a_1 r^{n-1}
\]
\[
a_n = -20(0.5)^{n-1}
\]
To find the seventh term of the sequence, let \( n = 7 \).
\[
a_n = -20(0.5)^{7-1}
\]
\[
= -20(0.5)^6
\]
\[
= -20(0.015625)
\]
\[
= -0.3125
\]
So, the seventh term is \(-0.3125\).
49. 4, −12, 36, …

**SOLUTION:**

The first term of the sequence is 4. So, \(a_1 = 4\).

Calculate the common ratio.

\[
\begin{align*}
4 & \quad -12 \quad 36 \\
\frac{-12}{4} & = -3 \\
\frac{36}{-12} & = -3
\end{align*}
\]

The common ratio is −3. So, \(r = -3\).

\[
a_n = a_1r^{n-1}
\]

\[
a_n = 4(-3)^{n-1}
\]

To find the seventh term of the sequence, let \(n = 7\).

\[
a_7 = 4(-3)^{7-1}
\]

\[
= 4(-3)^6
\]

\[
= 4(729)
\]

\[
= 2916
\]

So, the seventh term is 2916.
50. 99, −33, 11, …

**SOLUTION:**
The first term of the sequence is 99. So, \( a_1 = 99 \).

Calculate the common ratio.

\[
\begin{array}{ccc}
99 & -33 & 11 \\
-33 & -1 & 11 \\
\frac{99}{-33} & \frac{-1}{3} & \frac{11}{-33} = -\frac{1}{3}
\end{array}
\]

The common ratio is \(-\frac{1}{3}\). So, \( r = -\frac{1}{3} \).

\[
a_n = a_1r^{n-1}
\]

\[
a_n = 99 \left( -\frac{1}{3} \right)^{n-1}
\]

To find the seventh term of the sequence, let \( n = 7 \).

\[
a_n = 99 \left( -\frac{1}{3} \right)^{6}
\]

\[
= 99 \left( \frac{1}{729} \right)
\]

\[
= \frac{99}{729}
\]

\[
= \frac{11}{81}
\]

So, the seventh term is \( \frac{11}{81} \).
9-6 Analyzing Functions with Successive Differences

51. 22, 44, 88, …

**SOLUTION:**
The first term of the sequence is 22. So, \( a_1 = 22 \).

Calculate the common ratio.

\[
\begin{align*}
22 &\quad 44 &\quad 88 \\
\frac{44}{22} &\quad = 2 &\quad \frac{88}{44} &\quad = 2
\end{align*}
\]

The common ratio is 2. So, \( r = 2 \).

\[
a_n = a_1 r^{n-1}
\]

\[
a_n = 22(2)^{n-1}
\]

To find the seventh term of the sequence, let \( n = 7 \).

\[
a_n = 22(2)^{7-1}
\]
\[
= 22(2)^6
\]
\[
= 22(64)
\]
\[
= 1408
\]

So, the seventh term is 1408.
52. \( \frac{2}{3}, \frac{1}{3}, \frac{1}{6}, ... \)

**SOLUTION:**

The first term of the sequence is \( \frac{2}{3} \). So, \( a_1 = \frac{2}{3} \).

Calculate the common ratio.

\[
\begin{align*}
\frac{\frac{2}{3}}{\frac{1}{3}} &= \frac{1}{2} \\
\frac{\frac{1}{3}}{\frac{1}{6}} &= \frac{1}{2}
\end{align*}
\]

The common ratio is 2. So, \( r = \frac{1}{2} \)

\[
a_n = a_1 r^{n-1}
\]

\[
a_n = \frac{2}{3} \left( \frac{1}{2} \right)^{n-1}
\]

To find the seventh term of the sequence, let \( n = 7 \).

\[
a_n = \frac{2}{3} \left( \frac{1}{2} \right)^6 = \frac{2}{3} \left( \frac{1}{64} \right) = \frac{2}{192} = \frac{1}{96}
\]

So, the seventh term is \( \frac{1}{96} \).
53. **CANOE RENTAL** To rent a canoe, you must pay a daily rate plus $10 per hour. Ilia and her friends rented a canoe for 3 hours and paid $45. Write a linear equation for the cost \( C \) of renting the canoe for \( h \) hours, and determine how much it cost to rent the canoe for 8 hours.

**SOLUTION:**
\[
C = 10h + b
\]
\[
45 = 10(3) + b
\]
\[
45 = 30 + b
\]
\[
15 = b
\]
The total cost of renting the canoe for \( h \) hours can be represented by the equation \( C = 10h + 15 \).
To find the cost to rent the canoe for 8 hours, let \( h = 8 \).
\[
C = 10h + 15
\]
\[
= 10(8) + 15
\]
\[
= 80 + 15
\]
\[
= 95
\]
So, it costs $95 to rent the canoe for 8 hours.

**Determine whether each equation is a linear equation. If so, write the equation in standard form.**

54. \( 3x = 5y \)

**SOLUTION:**
Rewrite the equation so that it appears in standard form.
\[
3x = 5y
\]
\[
3x - 5y = 5y - 5y
\]
\[
3x - 5y = 0
\]
The equation is now in standard form where \( A = 3, B = -5, \) and \( C = 0 \). This is a linear equation.

55. \( 6 - y = 2x \)

**SOLUTION:**
Rewrite the equation so that it appears in standard form.
\[
6 - y = 2x
\]
\[
6 - y + y = 2x + y
\]
\[
6 = 2x + y
\]
The equation is now in standard form where \( A = 2, B = 1, \) and \( C = 6 \). This is a linear equation.

56. \( 6xy + 3x = 4 \)

**SOLUTION:**
Because one of the terms in the equation has two variables, the equation is not a linear equation.
9-6 Analyzing Functions with Successive Differences

57. \( y + 5 = 0 \)

**SOLUTION:**
Rewrite the equation so that it appears in standard form.
\( y + 5 = 0 \)
\( y + 5 - 5 = 0 - 5 \)
\( y = -5 \)
The equation is now in standard form where \( A = 0, B = 1, \) and \( C = -5. \) This is a linear equation.

58. \( 7y = 2x + 5x \)

**SOLUTION:**
Rewrite the equation so that it appears in standard form.
\( 7y = 2x + 5x \)
\( 7y = 7x \)
\( 7y - 7y = 7x - 7y \)
\( 0 = 7x - 7y \)
\( 0 = x - y \)
The equation is now in standard form where \( A = 1, B = -1, \) and \( C = 0. \) This is a linear equation.

59. \( y = 4x^2 - 1 \)

**SOLUTION:**
Because one of the terms in the equation has an exponent of 2, the equation is not a linear equation.

**Evaluate each expression if \( x = -3, y = -1, \) and \( z = 4. \)**

60. \( |x - 4| \)

**SOLUTION:**
\( |x - 4| = |-3 - 4| \)
\( = |-7| \)
\( = 7 \)

61. \( |2y + 1| \)

**SOLUTION:**
\( |2y + 1| = |2(-1) + 1| \)
\( = |-2 + 1| \)
\( = |-1| \)
\( = 1 \)

62. \( |4 - z| \)

**SOLUTION:**
\( |4 - z| = |4 - 4| \)
\( = |0| \)
\( = 0 \)
63. \(\left| \frac{1}{2}x + 2 \right|\)

**SOLUTION:**
\[
\left| \frac{1}{2}x + 2 \right| = \left| \frac{1}{2}(-3) + 2 \right| \\
= \left| -\frac{3}{2} + 2 \right| \\
= \left| \frac{1}{2} \right| \\
= \frac{1}{2}
\]

64. \(\left| 12 - 4z \right|\)

**SOLUTION:**
\[
\left| 12 - 4z \right| = |12 - 4(4)| \\
= |12 - 16| \\
= |-4| \\
= 4
\]

65. \(\left| 2y - 3 \right| - 6\)

**SOLUTION:**
\[
\left| 2y - 3 \right| - 6 = |2(-1) - 3| - 6 \\
= |-2 - 3| - 6 \\
= |-5| - 6 \\
= 5 - 6 \\
= -1
\]