9-1 Graphing Quadratic Functions

Use a table of values to graph each equation. State the domain and range.

1. \( y = 2x^2 + 4x - 6 \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 2x^2 + 4x - 6 )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>( y = 2(-3)^2 + 4(-3) - 6 = 0 )</td>
<td>(-3,0)</td>
</tr>
<tr>
<td>-2</td>
<td>( y = 2(-2)^2 + 4(-2) - 6 = -6 )</td>
<td>(-2,-6)</td>
</tr>
<tr>
<td>-1</td>
<td>( y = 2(-1)^2 + 4(-1) - 6 = -8 )</td>
<td>(-1,-8)</td>
</tr>
<tr>
<td>0</td>
<td>( y = 2(0)^2 + 4(0) - 6 = -6 )</td>
<td>(0,-6)</td>
</tr>
<tr>
<td>1</td>
<td>( y = 2(1)^2 + 4(1) - 6 = 0 )</td>
<td>(1,0)</td>
</tr>
<tr>
<td>2</td>
<td>( y = 2(2)^2 + 4(2) - 6 = 10 )</td>
<td>(2,10)</td>
</tr>
</tbody>
</table>

Graph the ordered pairs, and connect them to create a smooth curve. The parabola extends to infinity.

The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or \( \{ y \mid y \geq -8 \} \).
9-1 Graphing Quadratic Functions

2. \( y = x^2 + 2x - 1 \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = x^2 + 2x - 1 )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>( y = (-3)^2 + 2(-3) - 1 = \frac{-1}{2} )</td>
<td>(-3,2)</td>
</tr>
<tr>
<td>-2</td>
<td>( y = (-2)^2 + 2(-2) - 1 = -1 )</td>
<td>(-2,-1)</td>
</tr>
<tr>
<td>-1</td>
<td>( y = (-1)^2 + 2(-1) - 1 = -2 )</td>
<td>(-1,-2)</td>
</tr>
<tr>
<td>0</td>
<td>( y = (0)^2 + 2(0) - 1 = -1 )</td>
<td>(0,-1)</td>
</tr>
<tr>
<td>1</td>
<td>( y = (1)^2 + 2(1) - 1 = 2 )</td>
<td>(1,2)</td>
</tr>
<tr>
<td>2</td>
<td>( y = (2)^2 + 2(2) - 1 = 7 )</td>
<td>(2,7)</td>
</tr>
</tbody>
</table>

Graph the ordered pairs, and connect them to create a smooth curve. The parabola extends to infinity.

![Graph of y = x^2 + 2x - 1](image)

The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or \( \{y \mid y \geq -2\} \).
3. \( y = x^2 - 6x - 3 \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = x^2 - 6x - 3 )</th>
<th>( (x,y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>((-1)^2 - 6(-1) - 3 = 4)</td>
<td>(-1,4)</td>
</tr>
<tr>
<td>0</td>
<td>((0)^2 - 6(0) - 3 = -3)</td>
<td>(0,-3)</td>
</tr>
<tr>
<td>1</td>
<td>((1)^2 - 6(1) - 3 = -8)</td>
<td>(1,-8)</td>
</tr>
<tr>
<td>2</td>
<td>((2)^2 - 6(2) - 3 = -11)</td>
<td>(2,-11)</td>
</tr>
<tr>
<td>3</td>
<td>((3)^2 - 6(3) - 3 = -12)</td>
<td>(3,-12)</td>
</tr>
<tr>
<td>4</td>
<td>((4)^2 - 6(4) - 3 = -11)</td>
<td>(4,-11)</td>
</tr>
<tr>
<td>5</td>
<td>((5)^2 - 6(5) - 3 = -8)</td>
<td>(5,-8)</td>
</tr>
<tr>
<td>6</td>
<td>((6)^2 - 6(6) - 3 = -3)</td>
<td>(6,-3)</td>
</tr>
<tr>
<td>7</td>
<td>((7)^2 - 6(7) - 3 = 4)</td>
<td>(7,4)</td>
</tr>
</tbody>
</table>

Graph the ordered pairs, and connect them to create a smooth curve. The parabola extends to infinity.

The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or \( \{y | y \geq -12\} \).
4. \( y = 3x^2 - 6x - 5 \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 3x^2 - 6x - 5 )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>( y = 3(-2)^2 - 6(-2) - 5 = \frac{19}{19} )</td>
<td>(-2,19)</td>
</tr>
<tr>
<td>-1</td>
<td>( y = 3(-1)^2 - 6(-1) - 5 = 4 )</td>
<td>(-1,4)</td>
</tr>
<tr>
<td>0</td>
<td>( y = 3(0)^2 - 6(0) - 5 = -5 )</td>
<td>(0,-5)</td>
</tr>
<tr>
<td>1</td>
<td>( y = 3(1)^2 - 6(1) - 5 = -8 )</td>
<td>(1,-8)</td>
</tr>
<tr>
<td>2</td>
<td>( y = 3(2)^2 - 6(2) - 5 = -5 )</td>
<td>(2,-5)</td>
</tr>
<tr>
<td>3</td>
<td>( y = 3(3)^2 - 6(3) - 5 = 4 )</td>
<td>(3,4)</td>
</tr>
</tbody>
</table>

Graph the ordered pairs, and connect them to create a smooth curve. The parabola extends to infinity.

The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or \( \{ y | y \geq -8 \} \).

Find the vertex, the equation of the axis of symmetry, and the y-intercept of each graph.

5.

**SOLUTION:**

**Find the vertex.**
Because the parabola opens down, the vertex is located at the maximum point of the parabola. It is located at \((-1, 5)\).

**Find the axis of symmetry.**
The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at \( x = -1 \).

**Find the y-intercept.**
The y-intercept is the point where the graph intersects the y-axis. It is located at \((0, 3)\), so the y-intercept is 3.
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6.

**SOLUTION:**

**Find the vertex.**

Because the parabola opens up, the vertex is located at the minimum point of the parabola. It is located at (−2, −3).

**Find the axis of symmetry.**

The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at \( x = −2 \).

**Find the y-intercept.**

The y-intercept is the point where the graph intersects the y-axis. It is located at (0, 1), so the y-intercept is 1.

7.

**SOLUTION:**

**Find the vertex.**

Because the parabola opens up, the vertex is located at the minimum point of the parabola. It is located at (−2, −12).

**Find the axis of symmetry.**

The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at \( x = −2 \).

**Find the y-intercept.**

The y-intercept is the point where the graph intersects the y-axis. It is located at (0, −4), so the y-intercept is −4.
9-1 Graphing Quadratic Functions

SOLUTION:
Find the vertex.
Because the parabola opens down, the vertex is located at the maximum point of the parabola. It is located at (0, 5).

Find the axis of symmetry.
The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at \( x = 0 \).

Find the y-intercept.
The y-intercept is the point where the graph intersects the y-axis. It is located at (0, 5), so the y-intercept is 5.

Find the vertex, the equation of the graph of the axis of symmetry, and the y-intercept of the graph of each function.

9. \( y = -3x^2 + 6x - 1 \)

SOLUTION:
Find the vertex.
In the equation \( y = -3x^2 + 6x - 1 \), \( a = -3 \), \( b = 6 \), and \( c = -1 \).

The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
x = \frac{-b}{2a} = \frac{-6}{-6} = 1
\]

The \( x \)-coordinate of the vertex is \( x = 1 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
f(x) = -3x^2 + 6x - 1
\]

\[
f(1) = -3(1)^2 + 6(1) - 1 = 2
\]

The vertex is at (1, 2).

Find the axis of symmetry.
The axis of symmetry is the vertical line that goes through the vertex. It is located at \( x = 1 \).

Find the y-intercept.
The y-intercept always occurs at (0, \( c \)). Since \( c = -1 \) for this equation, the y-intercept is located at (0, -1).
10. \( y = -x^2 + 2x + 1 \)

**SOLUTION:**

**Find the vertex.**

In the equation \( y = -x^2 + 2x + 1 \), \( a = -1 \), \( b = 2 \), and \( c = 1 \).

The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
x = \frac{-b}{2a} = \frac{-2}{2 \cdot (-1)} = \frac{-2}{-2}
\]

\( x = 1 \)

The \( x \)-coordinate of the vertex is \( x = 1 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
f(x) = -x^2 + 2x + 1
\]

\[
f(1) = -(1)^2 + 2(1) + 1
\]

\[
f(1) = -1 + 2 + 1
\]

\( f(1) = 2 \)

The vertex is at \( (1, 2) \).

**Find the axis of symmetry.**

The axis of symmetry is the vertical line that goes through the vertex. It is located at \( x = 1 \).

**Find the \( y \)-intercept.**

The \( y \)-intercept always occurs at \( (0, c) \). Since \( c = 1 \) for this equation, the \( y \)-intercept is located at \( (0, 1) \).
9-1 Graphing Quadratic Functions

11. \( y = x^2 - 4x + 5 \)

**SOLUTION:**
Find the vertex.
In the equation \( y = x^2 - 4x + 5 \), \( a = 1 \), \( b = -4 \), and \( c = 5 \).
The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
x = \frac{-(-4)}{2 \cdot 1} = \frac{4}{2} = 2
\]

The \( x \)-coordinate of the vertex is \( x = 2 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
f(x) = x^2 - 4x + 5
\]

\[
f(2) = (2)^2 - 4(2) + 5 = 4 - 8 + 5 = 1
\]

The vertex is at \((2, 1)\).
Find the axis of symmetry.
The axis of symmetry is the vertical line that goes through the vertex. It is located at \( x = 2 \).

Find the \( y \)-intercept.
The \( y \)-intercept always occurs at \((0, c)\). Since \( c = 5 \) for this equation, the \( y \)-intercept is located at \((0, 5)\).
12. \( y = 4x^2 - 8x + 9 \)

**SOLUTION:**

**Find the vertex.**

In the equation \( y = 4x^2 - 8x + 9 \), \( a = 4 \), \( b = -8 \), and \( c = 9 \).

The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
x = \frac{-(-8)}{2 \cdot 4}
\]

\[
x = \frac{8}{8}
\]

\[
x = 1
\]

The \( x \)-coordinate of the vertex is \( x = 1 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
f(x) = 4x^2 - 8x + 9
\]

\[
f(1) = 4(1)^2 - 8(1) + 9
\]

\[
f(1) = 4 - 8 + 9
\]

\[
f(1) = 5
\]

The vertex is at \((1, 5)\).

**Find the axis of symmetry.**

The axis of symmetry is the vertical line that goes through the vertex. It is located at \( x = 1 \).

**Find the \( y \)-intercept.**

The \( y \)-intercept always occurs at \((0, c)\). Since \( c = 9 \) for this equation, the \( y \)-intercept is located at \((0, 9)\).
9-1 Graphing Quadratic Functions

Consider each function.

a. Determine whether the function has maximum or minimum value.

b. State the maximum or minimum value.

c. What are the domain and range of the function?

13. \(y = -x^2 + 4x - 3\)

**SOLUTION:**

a. For \(y = -x^2 + 4x - 3\), \(a = -1\), \(b = 4\), and \(c = -3\). Because \(a\) is negative, the graph opens downward, so the function has a maximum value.

b. The maximum value is the \(y\)-coordinate of the vertex. The \(x\)-coordinate of the vertex is \(x = \frac{-b}{2a}\).

\[
x = \frac{-b}{2a} = \frac{-4}{2 \cdot (-1)} = \frac{-4}{-2} = 2
\]

The \(x\)-coordinate of the vertex is \(x = 2\). Substitute this value into the function to find the \(y\)-coordinate.

\[
f(x) = -x^2 + 4x - 3
\]

\[
f(2) = -(2)^2 + 4(2) - 3 = -4 + 8 - 3 = 1
\]

The maximum value is 1.

c. The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or \(\{f(x) : f(x) \leq 1\}\).
14. \( y = -x^2 - 2x + 2 \)

**SOLUTION:**

a. For \( y = -x^2 - 2x + 2 \), \( a = -1 \), \( b = -2 \), and \( c = 2 \). Because \( a \) is negative, the graph opens downward, so the function has a maximum value.

b. The maximum value is the \( y \)-coordinate of the vertex. The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
x = \frac{-b}{2a} = \frac{-(-2)}{2 \cdot (-1)} = \frac{2}{-2} = -1
\]

The \( x \)-coordinate of the vertex is \( x = -1 \). Substitute this value into the function to find the \( y \)-coordinate.

\[
f(-1) = -(-1)^2 - 2(-1) + 2 = -1 + 2 + 2 = 3
\]

The maximum value is 3.

c. The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or \( \{ f(x) \mid f(x) \leq 3 \} \).

15. \( y = -3x^2 + 6x + 3 \)

**SOLUTION:**

a. For \( y = -3x^2 + 6x + 3 \), \( a = -3 \), \( b = 6 \), and \( c = 3 \). Because \( a \) is negative, the graph opens downward, so the function has a maximum value.

b. The maximum value is the \( y \)-coordinate of the vertex. The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

The \( x \)-coordinate of the vertex is \( x = 1 \). Substitute this value into the function to find the \( y \)-coordinate.

\[
f(1) = -3(1)^2 + 6(1) + 3 = -3 + 6 + 3 = 6
\]

The maximum value is 6.

c. The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or \( \{ f(x) \mid f(x) \leq 6 \} \).
9-1 Graphing Quadratic Functions

16. \( y = -2x^2 + 8x - 6 \)

**SOLUTION:**

a. For \( y = -2x^2 + 8x - 6 \), \( a = -2 \), \( b = 8 \), and \( c = -6 \). Because \( a \) is negative, the graph opens downward, so the function has a maximum value.

b. The maximum value is the \( y \)-coordinate of the vertex. The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
x = \frac{-b}{2a} \\
x = \frac{-8}{2 \cdot (-2)} \\
x = \frac{-8}{-4} \\
x = 2.
\]

The \( x \)-coordinate of the vertex is \( x = 2 \). Substitute this value into the function to find the \( y \)-coordinate.

\[
f(x) = -2x^2 + 8x - 6 \\
f(2) = -2(2)^2 + 8(2) - 6 \\
f(2) = -8 + 16 - 6 \\
f(2) = 2
\]

The maximum value is 2.

c. The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or \( \{ f(x) \mid f(x) \leq 2 \} \).
Graph each function.

17. \( f(x) = -3x^2 + 6x + 3 \)

**SOLUTION:**

**Step 1** Find the equation of the axis of symmetry. For \( f(x) = -3x^2 + 6x + 3 \), \( a = -3 \), \( b = 6 \), and \( c = 3 \).

\[
\begin{align*}
\text{vertex} & = \frac{-b}{2a} \\
& = \frac{-6}{2(-3)} \\
& = 1
\end{align*}
\]

**Step 2** Find the vertex, and determine whether it is a maximum or minimum.
The \( x \)-coordinate of the vertex is \( x = 1 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
\begin{align*}
f(x) &= -3x^2 + 6x + 3 \\
f(1) &= -3(1)^2 + 6(1) + 3 \\
f(1) &= -3 + 6 + 3 \\
f(1) &= 6
\end{align*}
\]
The vertex lies at \( (1, 6) \). Because \( a \) is negative, the graph opens down, and the vertex is a maximum.

**Step 3** Find the \( y \)-intercept.
Use the original equation, and substitute 0 for \( x \).

\[
\begin{align*}
y &= -3x^2 + 6x + 3 \\
y &= -3(0)^2 + 6(0) + 3 \\
y &= 0 + 0 + 3 \\
y &= 3
\end{align*}
\]
The \( y \)-intercept is \( (0, 3) \).

**Step 4** The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same \( y \)-value.

| \( x \) | -1 | 0 | 1 | 2 | 3 |
| \( y \) | -6 | 3 | 6 | 3 | -6 |

**Step 5** Connect the points with a smooth curve.
9-1 Graphing Quadratic Functions

18. \( f(x) = -2x^2 + 4x + 1 \)

**SOLUTION:**

**Step 1** Find the equation of the axis of symmetry. For \( f(x) = -2x^2 + 4x + 1 \), \( a = -2 \), \( b = 4 \), and \( c = 1 \).

\[
x = \frac{-b}{2a}
\]

\[
x = \frac{-4}{2(-2)}
\]

\[
x = \frac{-4}{-4}
\]

\[
x = 1
\]

**Step 2** Find the vertex, and determine whether it is a maximum or minimum.

The \( x \)-coordinate of the vertex is \( x = 1 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
f(x) = -2x^2 + 4x + 1
\]

\[
f(1) = -2(1)^2 + 4(1) + 1
\]

\[
f(1) = -2 + 4 + 1
\]

\[
f(1) = 3
\]

The vertex lies at (1, 3). Because \( a \) is negative, the graph opens down, and the vertex is a maximum.

**Step 3** Find the \( y \)-intercept.

Use the original equation, and substitute 0 for \( x \).

\[
y = -2x^2 + 4x + 1
\]

\[
y = -2(0)^2 + 4(0) + 1
\]

\[
y = 0 + 0 + 1
\]

\[
y = 1
\]

The \( y \)-intercept is (0, 1).

**Step 4** The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same \( y \)-value.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-5</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>-5</td>
</tr>
</tbody>
</table>

**Step 5** Connect the points with a smooth curve.
9-1 Graphing Quadratic Functions

19. \[ f(x) = 2x^2 - 8x - 4 \]

\textbf{SOLUTION:}

\textbf{Step 1} Find the equation of the axis of symmetry. For \( f(x) = 2x^2 - 8x - 4 \), \( a = 2 \), \( b = -8 \), and \( c = -4 \).

\[ x = \frac{-b}{2a} \]

\[ x = \frac{-(-8)}{2(2)} \]

\[ x = \frac{8}{4} \]

\[ x = 2 \]

\textbf{Step 2} Find the vertex, and determine whether it is a maximum or minimum. The \( x \)-coordinate of the vertex is \( x = 2 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[ f(x) = 2x^2 - 8x - 4 \]

\[ f(2) = 2(2)^2 - 8(2) - 4 \]

\[ f(2) = 8 - 16 - 4 \]

\[ f(2) = -12 \]

The vertex lies at \((2, -12)\). Because \( a \) is positive, the graph opens up, and the vertex is a minimum.

\textbf{Step 3} Find the \( y \)-intercept. Use the original equation, and substitute 0 for \( x \).

\[ y = 2x^2 - 8x - 4 \]

\[ y = 2(0)^2 - 8(0) - 4 \]

\[ y = 0 + 0 - 4 \]

\[ y = -4 \]

The \( y \)-intercept is \((0, -4)\).

\textbf{Step 4} The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same \( y \)-value.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-4</td>
<td>-10</td>
<td>-12</td>
<td>-10</td>
<td>-4</td>
</tr>
</tbody>
</table>

\textbf{Step 5} Connect the points with a smooth curve.
20. \( f(x) = 3x^2 - 6x - 1 \)

**SOLUTION:**

**Step 1** Find the equation of the axis of symmetry. For \( f(x) = 3x^2 - 6x - 1 \), \( a = 3 \), \( b = -6 \), and \( c = -1 \).

\[
x = \frac{-b}{2a}
\]

\[
x = \frac{-(-6)}{2 \cdot 3} = \frac{6}{6} = 1
\]

**Step 2** Find the vertex, and determine whether it is a maximum or minimum.

The \( x \)-coordinate of the vertex is \( x = 1 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
f(x) = 3x^2 - 6x - 1
\]

\[
f(1) = 3(1)^2 - 6(1) - 1
\]

\[
f(1) = 3 - 6 - 1
\]

\[
f(1) = -4
\]

The vertex lies at \((1, -4)\). Because \( a \) is positive, the graph opens up, and the vertex is a minimum.

**Step 3** Find the \( y \)-intercept.

Use the original equation, and substitute 0 for \( x \).

\[
y = 3x^2 - 6x - 1
\]

\[
y = 3(0)^2 - 6(0) - 1
\]

\[
y = 0 - 0 - 1
\]

\[
y = -1
\]

The \( y \)-intercept is \((0, -1)\).

**Step 4** The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same \( y \)-value.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>-1</td>
<td>-1</td>
<td>-4</td>
<td>-1</td>
<td>8</td>
</tr>
<tr>
<td>( y )</td>
<td>8</td>
<td>-1</td>
<td>4</td>
<td>-1</td>
<td>8</td>
</tr>
</tbody>
</table>

**Step 5** Connect the points with a smooth curve.

---

21. **CCSS REASONING** A juggler is tossing a ball into the air. The height of the ball in feet can be modeled by the
9-1 Graphing Quadratic Functions

equation \( y = -16x^2 + 16x + 5 \), where \( y \) represents the height of the ball at \( x \) seconds.

a. Graph this equation.
b. At what height is the ball thrown?
c. What is the maximum height of the ball?

**SOLUTION:**

a. **Step 1** Find the equation of the axis of symmetry. For \( y = -16x^2 + 16x + 5 \), \( a = -16 \), \( b = 16 \), and \( c = 5 \).

\[
x = \frac{-b}{2a} \\
x = \frac{-16}{2 \cdot (-16)} \\
x = -\frac{16}{-32} \\
x = \frac{1}{2}
\]

**Step 2** Find the vertex, and determine whether it is a maximum or minimum.

The \( x \)-coordinate of the vertex is \( x = \frac{1}{2} \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
y = -16\left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{2}\right) + 5 \\
y = -16\left(\frac{1}{4}\right) + 8 + 5 \\
y = -4 + 8 + 5 \\
y = 9
\]

The vertex lies at \( \left(\frac{1}{2}, 9\right) \). Because \( a \) is negative, the graph opens down, and the vertex is a maximum.

**Step 3** Find the \( y \)-intercept.

Use the original equation, and substitute 0 for \( x \).

\[
y = -16x^2 + 16x + 5 \\
y = -16(0)^2 + 16(0) + 5 \\
y = 0 + 0 + 5 \\
y = 5
\]

The \( y \)-intercept is \( (0, 5) \).

**Step 4** The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same \( y \)-value.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>( \frac{1}{2} )</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5</td>
<td>9</td>
<td>5</td>
</tr>
</tbody>
</table>

**Step 5** Connect the points with a smooth curve.
9-1 Graphing Quadratic Functions

b. The ball is thrown when time equals 0, or at the y-intercept. So, the ball was 5 feet from the ground when it was thrown.
c. The maximum height of the ball occurs at the vertex. So, the ball reaches a maximum height of 9 feet.

Use a table of values to graph each equation. State the domain and range.

22. \( y = x^2 + 4x + 6 \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = x^2 + 4x + 6 )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>( y = (-4)^2 + 4(-4) + 6 = 6 )</td>
<td>(-4,6)</td>
</tr>
<tr>
<td>-3</td>
<td>( y = (-3)^2 + 4(-3) + 6 = 3 )</td>
<td>(-3,-3)</td>
</tr>
<tr>
<td>-2</td>
<td>( y = (-2)^2 + 4(-2) + 6 = 2 )</td>
<td>(-2,2)</td>
</tr>
<tr>
<td>-1</td>
<td>( y = (-1)^2 + 4(-1) + 6 = 3 )</td>
<td>(-1,3)</td>
</tr>
<tr>
<td>0</td>
<td>( y = (0)^2 + 4(0) + 6 = 6 )</td>
<td>(0,6)</td>
</tr>
</tbody>
</table>

Graph the ordered pairs, and connect them to create a smooth curve. The parabola extends to infinity.

The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or \( \{y | y \geq 2 \} \).
23. \( y = 2x^2 + 4x + 7 \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 2x^2 + 4x + 7 )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>2(-3)^2 + 4(-3) + 7 = 13</td>
<td>(-3, 13)</td>
</tr>
<tr>
<td>-2</td>
<td>2(-2)^2 + 4(-2) + 7 = 7</td>
<td>(-2, 7 )</td>
</tr>
<tr>
<td>-1</td>
<td>2(-1)^2 + 4(-1) + 7 = 5</td>
<td>(-1, 5 )</td>
</tr>
<tr>
<td>0</td>
<td>2(0)^2 + 4(0) + 7 = 7</td>
<td>(0, 7 )</td>
</tr>
<tr>
<td>1</td>
<td>2(1)^2 + 4(1) + 7 = 13</td>
<td>(1, 13)</td>
</tr>
</tbody>
</table>

Graph the ordered pairs, and connect them to create a smooth curve. The parabola extends to infinity.

The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or \( \{y \mid y \geq -5\} \).

24. \( y = 2x^2 - 8x - 5 \)

**SOLUTION:**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 2x^2 - 8x - 5 )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2(4)^2 - 8(4) - 5 = -5</td>
<td>(4, -5 )</td>
</tr>
<tr>
<td>3</td>
<td>2(3)^2 - 8(3) - 5 = -11</td>
<td>(3, -11 )</td>
</tr>
<tr>
<td>2</td>
<td>2(2)^2 - 8(2) - 5 = -13</td>
<td>(2, -13 )</td>
</tr>
<tr>
<td>1</td>
<td>2(1)^2 - 8(1) - 5 = -11</td>
<td>(1, -13 )</td>
</tr>
<tr>
<td>0</td>
<td>2(0)^2 - 8(0) - 5 = -5</td>
<td>(0, -5 )</td>
</tr>
</tbody>
</table>

Graph the ordered pairs, and connect them to create a smooth curve. The parabola extends to infinity.

The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or \( \{y \mid y \geq -13\} \).
25. $y = 3x^2 + 12x + 5$

**SOLUTION:**

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = 3x^2 + 12x + 5$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$y = 3(0)^2 + 12(0) + 5 = 5$</td>
<td>(0, 5)</td>
</tr>
<tr>
<td>−1</td>
<td>$y = 3(-1)^2 + 12(-1) + 5 = -4$</td>
<td>(−1, −4)</td>
</tr>
<tr>
<td>−2</td>
<td>$y = 3(-2)^2 + 12(-2) + 5 = -7$</td>
<td>(−2, −7)</td>
</tr>
<tr>
<td>−3</td>
<td>$y = 3(-3)^2 + 12(-3) + 5 = -4$</td>
<td>(−3, −4)</td>
</tr>
<tr>
<td>−4</td>
<td>$y = 3(-4)^2 + 12(-4) + 5 = 5$</td>
<td>(−4, 5)</td>
</tr>
</tbody>
</table>

Graph the ordered pairs, and connect them to create a smooth curve. The parabola extends to infinity.

The domain of the equation is all real numbers. The range of the equation is all real numbers greater than or equal to the minimum value, or \( \{ y \mid y \geq -7 \} \).

26. $y = 3x^2 - 6x - 2$

**SOLUTION:**

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = 3x^2 - 6x - 2$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$y = 3(0)^2 - 6(0) - 2 = -2$</td>
<td>(0, −2)</td>
</tr>
<tr>
<td>1</td>
<td>$y = 3(1)^2 - 6(1) - 2 = -5$</td>
<td>(1, −5)</td>
</tr>
<tr>
<td>2</td>
<td>$y = 3(2)^2 - 6(2) - 2 = -2$</td>
<td>(2, −2)</td>
</tr>
<tr>
<td>3</td>
<td>$y = 3(3)^2 - 6(3) - 2 = 7$</td>
<td>(3, 7)</td>
</tr>
<tr>
<td>−1</td>
<td>$y = 3(-1)^2 - 6(-1) - 2 = 7$</td>
<td>(−1, 7)</td>
</tr>
</tbody>
</table>

Graph the ordered pairs, and connect them to create a smooth curve. The parabola extends to infinity.

The domain of the equation is all real numbers. The range of the equation is all real numbers greater than or equal to the minimum value, or \( \{ y \mid y \geq -5 \} \).
9-1 Graphing Quadratic Functions

27. \( y = x^2 - 2x - 1 \)

**SOLUTION:**

\[
\begin{array}{c|c|c}
 x & y = x^2 - 2x - 1 & (x, y) \\
\hline
 3 & y = (3)^2 - 2(3) - 1 = 2 & (3, 2) \\
 2 & y = (2)^2 - 2(2) - 1 = -1 & (2, -1) \\
 1 & y = (1)^2 - 2(1) - 1 = -2 & (1, -2) \\
 0 & y = (0)^2 - 2(0) - 1 = -1 & (0, -1) \\
-1 & y = (-1)^2 - 2(-1) - 1 = 2 & (-1, 2) \\
\end{array}
\]

Graph the ordered pairs, and connect them to create a smooth curve. The parabola extends to infinity.

The domain of the equation is all real numbers. The range of the equation is all real numbers greater than or equal to the minimum value, or \( \{ y | y \geq -2 \} \).

**Find the vertex, the equation of the axis of symmetry, and the y-intercept of each graph.**

**SOLUTION:**

**Find the vertex.**

Because the parabola opens up, the vertex is located at the minimum point of the parabola. It is located at \((-3, -6)\).

**Find the axis of symmetry.**

The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at \(x = -3\).

**Find the y-intercept.**

The y-intercept is the point where the graph intersects the y-axis. It is located at \((0, 3)\), so the y-intercept is 3.
SOLUTION:
Find the vertex.
Because the parabola opens down, the vertex is located at the maximum point of the parabola. It is located at (0, 1).
Find the axis of symmetry.
The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at \( x = 0 \).
Find the \( y \)-intercept.
The \( y \)-intercept is the point where the graph intersects the \( y \)-axis. It is located at (0, 1), so the \( y \)-intercept is 1.

SOLUTION:
Find the vertex.
Because the parabola opens down, the vertex is located at the maximum point of the parabola. It is located at (−1, 5).
Find the axis of symmetry.
The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at \( x = −1 \).
Find the \( y \)-intercept.
The \( y \)-intercept is the point where the graph intersects the \( y \)-axis. It is located at (0, 4), so the \( y \)-intercept is 4.
9-1 Graphing Quadratic Functions

**SOLUTION:**

**Find the vertex.**
Because the parabola opens up, the vertex is located at the minimum point of the parabola. It is located at (1, 1).

**Find the axis of symmetry.**
The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at \( x = 1 \).

**Find the y-intercept.**
The y-intercept is the point where the graph intersects the y-axis. It is located at (0, 4), so the y-intercept is 4.

**SOLUTION:**

**Find the vertex.**
Because the parabola opens up, the vertex is located at the minimum point of the parabola. It is located at (0, –4).

**Find the axis of symmetry.**
The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at \( x = 0 \).

**Find the y-intercept.**
The y-intercept is the point where the graph intersects the y-axis. It is located at (0, –4), so the y-intercept is –4.
33. \[ y = 2x^2 + 4x - 6 \]

SOLUTION:
Find the vertex.
Because the parabola opens down, the vertex is located at the maximum point of the parabola. It is located at (0, 0).
Find the axis of symmetry.
The axis of symmetry is the line that goes through the vertex and divides the parabola into congruent halves. It is located at \( x = 0 \).
Find the y-intercept.
The y-intercept is the point where the graph intersects the y-axis. It is located at (0, 0), so the y-intercept is 0.

Find the vertex, the equation of the axis of symmetry, and the y-intercept of each function.

34. \( y = x^2 + 8x + 10 \)

SOLUTION:
Find the vertex.
In the equation \( y = x^2 + 8x + 10 \), \( a = 1 \), \( b = 8 \), and \( c = 10 \).
The x-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
\begin{align*}
x &= \frac{-b}{2a} \\
x &= \frac{-8}{2} \\
x &= -4
\end{align*}
\]

The x-coordinate of the vertex is \( x = -4 \). Substitute the x-coordinate of the vertex into the original equation to find the value of y.

\[
\begin{align*}
f(x) &= x^2 + 8x + 10 \\
f(-4) &= (-4)^2 + 8(-4) + 10 \\
f(-4) &= 16 - 32 + 10 \\
f(-4) &= -6
\end{align*}
\]
The vertex is at \((-4, -6)\).
Find the axis of symmetry.
The axis of symmetry is the vertical line that goes through the vertex. It is located at \( x = -4 \).
Find the y-intercept.
The y-intercept always occurs at \((0, c)\). Since \( c = 10 \) for this equation, the y-intercept is located at \((0, 10)\).
35. \( y = 2x^2 + 12x + 10 \)

**SOLUTION:**

**Find the vertex.**

In the equation \( y = 2x^2 + 12x + 10 \), \( a = 2 \), \( b = 12 \), and \( c = 10 \).

The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
x = \frac{-b}{2a} = \frac{-12}{2 \cdot 2} = \frac{-12}{4} = -3
\]

The \( x \)-coordinate of the vertex is \( x = -3 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
f(x) = 2x^2 + 12x + 10
\]

\[
f(-3) = 2(-3)^2 + 12(-3) + 10
\]

\[
f(-3) = 18 - 36 + 10
\]

\[
f(-3) = -8
\]

The vertex is at \((-3, -8)\).

**Find the axis of symmetry.**

The axis of symmetry is the vertical line that goes through the vertex. It is located at \( x = -3 \).

**Find the \( y \)-intercept.**

The \( y \)-intercept always occurs at \((0, c)\). Since \( c = 10 \) for this equation, the \( y \)-intercept is located at \((0, 10)\).
36. \( y = -3x^2 - 6x + 7 \)

**SOLUTION:**

Find the vertex.

In the equation \( y = -3x^2 - 6x + 7 \), \( a = -3 \), \( b = -6 \), and \( c = 7 \).

The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
x = \frac{-b}{2a} = \frac{-(6)}{2 \cdot (-3)} = \frac{6}{6} = 1
\]

The \( x \)-coordinate of the vertex is \( x = 1 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
f(x) = -3x^2 - 6x + 7
\]

\[
f(1) = -3(1)^2 - 6(1) + 7
\]

\[
f(1) = -3 + 6 + 7 = 10
\]

The vertex is at \((1, 10)\).

Find the axis of symmetry.

The axis of symmetry is the vertical line that goes through the vertex. It is located at \( x = -1 \).

Find the \( y \)-intercept.

The \( y \)-intercept always occurs at \((0, c)\). Since \( c = 7 \) for this equation, the \( y \)-intercept is located at \((0, 7)\).
37. \( y = -x^2 - 6x - 5 \)

**SOLUTION:**

Find the vertex.

In the equation \( y = -x^2 - 6x - 5 \), \( a = -1 \), \( b = -6 \), and \( c = -5 \).

The x-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
x = \frac{-b}{2a} = \frac{-(6)}{2 \times (-1)} = \frac{6}{2} = 3
\]

The x-coordinate of the vertex is \( x = -3 \). Substitute the x-coordinate of the vertex into the original equation to find the value of \( y \).

\[
f(x) = -x^2 - 6x - 5
\]

\[
f(-3) = -(-3)^2 - 6(-3) - 5 = -9 + 18 - 5 = 4
\]

The vertex is at \((-3, 4)\).

Find the axis of symmetry.

The axis of symmetry is the vertical line that goes through the vertex. It is located at \( x = -3 \).

Find the y-intercept.

The y-intercept always occurs at \((0, c)\). Since \( c = -5 \) for this equation, the y-intercept is located at \((0, -5)\).
9-1 Graphing Quadratic Functions

38. \( y = 5x^2 + 20x + 10 \)

**SOLUTION:**

Find the vertex.

In the equation \( y = 5x^2 + 20x + 10 \), \( a = 5 \), \( b = 20 \), and \( c = 10 \).

The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
\begin{align*}
x &= \frac{-b}{2a} \\
x &= \frac{-20}{2 \cdot 5} \\
x &= \frac{-20}{10} \\
x &= -2
\end{align*}
\]

The \( x \)-coordinate of the vertex is \( x = -2 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
\begin{align*}
f(x) &= 5x^2 + 20x + 10 \\
f(-2) &= 5(-2)^2 + 20(-2) + 10 \\
f(-2) &= 20 - 40 + 10 \\
f(-2) &= -10
\end{align*}
\]

The vertex is at \((-2, -10)\).

Find the axis of symmetry.

The axis of symmetry is the vertical line that goes through the vertex. It is located at \( x = -2 \).

Find the \( y \)-intercept.

The \( y \)-intercept always occurs at \((0, c)\). Since \( c = 10 \) for this equation, the \( y \)-intercept is located at \((0, 10)\).
39. \( y = 7x^2 - 28x + 14 \)

**SOLUTION:**

**Find the vertex.**

In the equation \( y = 7x^2 - 28x + 14 \), \( a = 7 \), \( b = -28 \), and \( c = 14 \).

The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
\begin{align*}
  x &= \frac{-(-28)}{2 \cdot 7} \\
  x &= \frac{28}{14} \\
  x &= 2
\end{align*}
\]

The \( x \)-coordinate of the vertex is \( x = 2 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
\begin{align*}
  f(x) &= 7x^2 - 28x + 14 \\
  f(2) &= 7(2)^2 - 28(2) + 14 \\
  f(2) &= 28 - 56 + 14 \\
  f(2) &= -14
\end{align*}
\]

The vertex is at \((2, -14)\).

**Find the axis of symmetry.**

The axis of symmetry is the vertical line that goes through the vertex. It is located at \( x = 2 \).

**Find the \( y \)-intercept.**

The \( y \)-intercept always occurs at \((0, c)\). Since \( c = 14 \) for this equation, the \( y \)-intercept is located at \((0, 14)\).
40. \( y = 2x^2 - 12x + 6 \)

**SOLUTION:**

Find the vertex.

In the equation \( y = 2x^2 - 12x + 6 \), \( a = 2 \), \( b = -12 \), and \( c = 6 \).

The x-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
x = \frac{-(-12)}{2 \cdot 2} = \frac{12}{4} = 3
\]

The x-coordinate of the vertex is \( x = 3 \). Substitute the x-coordinate of the vertex into the original equation to find the value of y.

\[
f(x) = 2x^2 - 12x + 6
\]

\[
f(3) = 2(3)^2 - 12(3) + 6 = 18 - 36 + 6 = -12
\]

The vertex is at \((3, -12)\).

Find the axis of symmetry.

The axis of symmetry is the vertical line that goes through the vertex. It is located at \( x = 3 \).

Find the y-intercept.

The y-intercept always occurs at \((0, c)\). Since \( c = 6 \) for this equation, the y-intercept is located at \((0, 6)\).
9-1 Graphing Quadratic Functions

41. \( y = -3x^2 + 6x - 18 \)

**SOLUTION:**

Find the vertex.

In the equation \( y = -3x^2 + 6x - 18 \), \( a = -3 \), \( b = 6 \), and \( c = -18 \).

The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
x = \frac{-b}{2a} = \frac{-6}{2(-3)} = \frac{-6}{-6} = 1
\]

The \( x \)-coordinate of the vertex is \( x = 1 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
f(x) = -3x^2 + 6x - 18
\]

\[
f(1) = -3(1)^2 + 6(1) - 18
f(1) = -3 + 6 - 18
f(1) = -15
\]

The vertex is at \((1, -15)\).

Find the axis of symmetry.

The axis of symmetry is the vertical line that goes through the vertex. It is located at \( x = 1 \).

Find the \( y \)-intercept.

The \( y \)-intercept always occurs at \((0, \ c)\). Since \( c = -18 \) for this equation, the \( y \)-intercept is located at \((0, -18)\).
42. $y = -x^2 + 10x - 13$

**SOLUTION:**

Find the vertex.

In the equation $y = -x^2 + 10x - 13$, $a = -1$, $b = 10$, and $c = -13$.

The $x$-coordinate of the vertex is $x = \frac{-b}{2a}$.

\[
x = \frac{-b}{2a} = \frac{-10}{2 \cdot (-1)} = \frac{-10}{-2} = 5
\]

The $x$-coordinate of the vertex is $x = 5$. Substitute the $x$-coordinate of the vertex into the original equation to find the value of $y$.

\[
f(x) = -x^2 + 10x - 13
\]

\[
f(5) = -(5)^2 + 10(5) - 13
\]

\[
f(5) = -25 + 50 - 13 = 12
\]

The vertex is at $(5, 12)$.

Find the axis of symmetry.

The axis of symmetry is the vertical line that goes through the vertex. It is located at $x = 5$.

Find the $y$-intercept.

The $y$-intercept always occurs at $(0, c)$. Since $c = -13$ for this equation, the $y$-intercept is located at $(0, -13)$. 

---

**9-1 Graphing Quadratic Functions**

Use a table of values to graph each equation. State the domain and range.

1. $y = 2x^2 + 4x - 6$

**SOLUTION:**

Graph the equation as shown in the diagram.

Find the vertex.

Find the axis of symmetry.

Find the $y$-intercept.

---

29. At what height is the ball hit?

---

30. e. State a reasonable range and domain for this situation.

**SOLUTION:**

maximum

---

31. It takes the boat 2 minutes to reach the vertex, and due to symmetry it will take the boat another 2 minutes to reach the other side.

---

32. \[y = -x^2 + 10x - 13\]

**SOLUTION:**

Find the vertex.

Find the $y$-intercept.

---

33. The vertex lies at \((0.857, 16.309)\).

---

34. The axis of symmetry is the vertical line that goes through the vertex. It is located at $x = 5$.

---

35. The vertex lies at \((-0.278, -0.162)\).

---

36. The vertex lies at \((0.6, 4.8)\).

---

37. The vertex lies at \((-0.6, -8.4)\).

---

38. The vertex lies at \((-0.7, -0.7)\).

---

39. The vertex lies at \((0, 5)\).

---
Consider each function.

a. Determine whether the function has a maximum or minimum value.

b. State the maximum or minimum value.

c. What are the domain and range of the function?

43. \( y = -2x^2 - 8x + 1 \)

SOLUTION:

a. For \( y = -2x^2 - 8x + 1 \), \( a = -2 \), \( b = -8 \), and \( c = 1 \). Because \( a \) is negative, the graph opens downward, so the function has a maximum value.

b. The maximum value is the \( y \)-coordinate of the vertex. The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
\begin{align*}
\frac{-b}{2a} & = \frac{8}{2(-2)} \\
\frac{-b}{2a} & = \frac{8}{-4} \\
\frac{-b}{2a} & = -2 \\
\end{align*}
\]

The \( x \)-coordinate of the vertex is \( x = -2 \). Substitute this value into the function to find the \( y \)-coordinate.

\[
\begin{align*}
f(x) & = -2x^2 - 8x + 1 \\
f(-2) & = -2(-2)^2 - 8(-2) + 1 \\
f(-2) & = -8 + 16 + 1 \\
f(-2) & = 9 \\
\end{align*}
\]

The maximum value is 9.

c. The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or \{\( f(x) \mid f(x) \leq 9 \} \).
44. \(y = x^2 + 4x - 5\)

**SOLUTION:**

a. For \(y = x^2 + 4x - 5\), \(a = 1\), \(b = 4\), and \(c = -5\). Because \(a\) is positive, the graph opens upward, so the function has a minimum value.

b. The minimum value is the \(y\)-coordinate of the vertex. The \(x\)-coordinate of the vertex is \(x = \frac{-b}{2a}\).

\[
\begin{align*}
\text{vertex} & = \left( -\frac{4}{2}, \frac{-12}{2} \right) \\
& = (-2, 6)
\end{align*}
\]

The \(x\)-coordinate of the vertex is \(x = -2\). Substitute this value into the function to find the \(y\)-coordinate.

\[
\begin{align*}
f(-2) & = (-2)^2 + 4(-2) - 5 \\
& = 4 - 8 - 5 \\
& = -9
\end{align*}
\]

The minimum value is \(-9\).

c. The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or \(f(x) \geq -9\).

45. \(y = 3x^2 + 18x - 21\)

**SOLUTION:**

a. For \(y = 3x^2 + 18x - 21\), \(a = 3\), \(b = 18\), and \(c = -21\). Because \(a\) is positive, the graph opens upward, so the function has a minimum value.

b. The minimum value is the \(y\)-coordinate of the vertex. The \(x\)-coordinate of the vertex is \(x = \frac{-b}{2a}\).

\[
\begin{align*}
\text{vertex} & = \left( -\frac{18}{2}, \frac{-54}{2} \right) \\
& = (-3, -9)
\end{align*}
\]

The \(x\)-coordinate of the vertex is \(x = -3\). Substitute this value into the function to find the \(y\)-coordinate.

\[
\begin{align*}
f(-3) & = 3(-3)^2 + 18(-3) - 21 \\
& = 27 - 54 - 21 \\
& = -48
\end{align*}
\]

The minimum value is \(-48\).

c. The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or \(f(x) \geq -48\).
9-1 Graphing Quadratic Functions

46. \( y = -2x^2 - 16x + 18 \)

**SOLUTION:**

a. For \( y = -2x^2 - 16x + 18 \), \( a = -2 \), \( b = -16 \), and \( c = 18 \). Because \( a \) is negative, the graph opens downward, so the function has a maximum value.

b. The maximum value is the \( y \)-coordinate of the vertex. The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
\begin{align*}
  x &= \frac{-b}{2a} \\
  x &= \frac{-(-16)}{2 \cdot (-2)} \\
  x &= \frac{16}{-4} \\
  x &= -4
\end{align*}
\]

The \( x \)-coordinate of the vertex is \( x = -4 \). Substitute this value into the function to find the \( y \)-coordinate.

\[
\begin{align*}
  f(x) &= -2x^2 - 16x + 18 \\
  f(-4) &= -2(-4)^2 - 16(-4) + 18 \\
  f(-4) &= -32 + 64 + 18 \\
  f(-4) &= 50
\end{align*}
\]

The maximum value is 50.

c. The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or \( \{f(x) \mid f(x) \leq 50\} \).
9-1 Graphing Quadratic Functions

47. \( y = -x^2 - 14x - 16 \)

**SOLUTION:**

a. For \( y = -x^2 - 14x - 16 \), \( a = -1 \), \( b = -14 \), and \( c = -16 \). Because \( a \) is negative, the graph opens downward, so the function has a maximum value.

b. The maximum value is the \( y \)-coordinate of the vertex. The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
\begin{align*}
x &= \frac{-b}{2a} \\
x &= \frac{-(-14)}{2 \cdot (-1)} \\
x &= \frac{14}{-2} \\
x &= -7
\end{align*}
\]

The \( x \)-coordinate of the vertex is \( x = -7 \). Substitute this value into the function to find the \( y \)-coordinate.

\[
\begin{align*}
f(x) &= -x^2 - 14x - 16 \\
f(-7) &= -(-7)^2 - 14(-7) - 16 \\
f(-7) &= 49 + 98 - 16 \\
f(-7) &= 133
\end{align*}
\]

The maximum value is 133.

c. The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or \( \{ f(x) \mid f(x) \leq 133 \} \).

48. \( y = 4x^2 + 40x + 44 \)

**SOLUTION:**

a. For \( y = 4x^2 + 40x + 44 \), \( a = 4 \), \( b = 40 \), and \( c = 44 \). Because \( a \) is positive, the graph opens upward, so the function has a minimum value.

b. The minimum value is the \( y \)-coordinate of the vertex. The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
\begin{align*}
x &= \frac{-b}{2a} \\
x &= \frac{-(40)}{2 \cdot 4} \\
x &= \frac{-40}{8} \\
x &= -5
\end{align*}
\]

The \( x \)-coordinate of the vertex is \( x = -5 \). Substitute this value into the function to find the \( y \)-coordinate.

\[
\begin{align*}
f(x) &= 4x^2 + 40x + 44 \\
f(-5) &= 4(-5)^2 + 40(-5) + 44 \\
f(-5) &= 100 - 200 + 44 \\
f(-5) &= -56
\end{align*}
\]

The minimum value is \(-56\).

c. The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or \( \{ f(x) \mid f(x) \geq -56 \} \).
9-1 Graphing Quadratic Functions

49. \( y = -x^2 - 6x - 5 \)

**SOLUTION:**

a. For \( y = -x^2 - 6x - 5 \), \( a = -1 \), \( b = -6 \), and \( c = -5 \). Because \( a \) is negative, the graph opens downward, so the function has a maximum value.

b. The maximum value is the \( y \)-coordinate of the vertex. The \( x \)-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
x = \frac{-b}{2a} = \frac{-(-6)}{2 \cdot (-1)} = \frac{6}{-2} = -3
\]

The \( x \)-coordinate of the vertex is \( x = -3 \). Substitute this value into the function to find the \( y \)-coordinate.

\[
f(x) = -x^2 - 6x - 5
\]

\[
f(-3) = -(-3)^2 - 6(-3) - 5 = -9 + 18 - 5 = 4
\]

The maximum value is 4.

c. The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or \( \{f(x) | f(x) \leq 4\} \).
50. \( y = 2x^2 + 4x + 6 \)

**SOLUTION:**

a. For \( y = 2x^2 + 4x + 6 \), \( a = 2 \), \( b = 4 \), and \( c = 6 \). Because \( a \) is positive, the graph opens upward, so the function has a minimum value.

b. The minimum value is the y-coordinate of the vertex. The x-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
\begin{align*}
x &= \frac{-b}{2a} \\
x &= \frac{-4}{2 \cdot 2} \\
x &= \frac{-4}{4} \\
x &= -1 \\
\end{align*}
\]

The x-coordinate of the vertex is \( x = -1 \). Substitute this value into the function to find the y-coordinate.

\[
\begin{align*}
f(x) &= 2x^2 + 4x + 6 \\
f(-1) &= 2(-1)^2 + 4(-1) + 6 \\
f(-1) &= 2 - 4 + 6 \\
f(-1) &= 4 \\
The minimum value is 4. \\
c. The domain is all real numbers. The range is all real numbers greater than or equal to the minimum value, or \( \{ f(x) | f(x) \geq 4 \} \).
\]

51. \( y = -3x^2 - 12x - 9 \)

**SOLUTION:**

a. For \( y = -3x^2 - 12x - 9 \), \( a = -3 \), \( b = -12 \), and \( c = -9 \). Because \( a \) is negative, the graph opens downward, so the function has a maximum value.

b. The maximum value is the y-coordinate of the vertex. The x-coordinate of the vertex is \( x = \frac{-b}{2a} \).

\[
\begin{align*}
x &= \frac{-b}{2a} \\
x &= \frac{-(-12)}{2 \cdot (-3)} \\
x &= \frac{12}{-6} \\
x &= -2 \\
\end{align*}
\]

The x-coordinate of the vertex is \( x = -2 \). Substitute this value into the function to find the y-coordinate.

\[
\begin{align*}
f(x) &= -3x^2 - 12x - 9 \\
f(-2) &= -3(-2)^2 - 12(-2) - 9 \\
f(-2) &= -12 + 24 - 9 \\
f(-2) &= 3 \\
The maximum value is 3. \\
c. The domain is all real numbers. The range is all real numbers less than or equal to the maximum value, or \( \{ f(x) | f(x) \leq 3 \} \).
9-1 Graphing Quadratic Functions

Graph each function.

52. \( y = -3x^2 + 6x - 4 \)

**SOLUTION:**

**Step 1** Find the equation of the axis of symmetry. For \( y = -3x^2 + 6x - 4 \), \( a = -3 \), \( b = 6 \), and \( c = -4 \).

\[
x = \frac{-b}{2a}
\]

\[
x = \frac{-6}{2 \cdot (-3)}
\]

\[
x = \frac{-6}{-6}
\]

\[
x = 1
\]

**Step 2** Find the vertex, and determine whether it is a maximum or minimum.

The \( x \)-coordinate of the vertex is \( x = 1 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
f(x) = -3x^2 + 6x - 4
\]

\[
f(1) = -3(1)^2 + 6(1) - 4
\]

\[
f(1) = -3 + 6 - 4
\]

\[
f(1) = -1
\]

The vertex lies at \((1, -1)\). Because \( a \) is negative, the graph opens down, and the vertex is a maximum.

**Step 3** Find the \( y \)-intercept.

Use the original equation, and substitute \( 0 \) for \( x \).

\[
y = -3x^2 + 6x - 4
\]

\[
y = -3(0)^2 + 6(0) - 4
\]

\[
y = 0 + 0 - 4
\]

\[
y = -4
\]

The \( y \)-intercept is \((0, -4)\).

**Step 4** The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same \( y \)-value.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-13</td>
<td>-4</td>
<td>-1</td>
<td>-4</td>
<td>-13</td>
</tr>
</tbody>
</table>

**Step 5** Connect the points with a smooth curve.
53. $y = -2x^2 - 4x - 3$

**SOLUTION:**

**Step 1** Find the equation of the axis of symmetry. For $y = -2x^2 - 4x - 3$, $a = -2$, $b = -4$, and $c = -3$.

$x = \frac{-b}{2a}$

$x = \frac{-(-4)}{2 \cdot (-2)}$

$x = \frac{4}{-4}$

$x = -1$

**Step 2** Find the vertex, and determine whether it is a maximum or minimum. The $x$-coordinate of the vertex is $x = -1$. Substitute the $x$-coordinate of the vertex into the original equation to find the value of $y$.

$f(x) = -2x^2 - 4x - 3$

$f(-1) = -2(-1)^2 - 4(-1) - 3$

$f(-1) = -2 + 4 - 3$

$f(-1) = -1$

The vertex lies at $(-1, -1)$. Because $a$ is negative, the graph opens down, and the vertex is a maximum.

**Step 3** Find the $y$-intercept. Use the original equation, and substitute 0 for $x$.

$y = -2x^2 - 4x - 3$

$y = -2(0)^2 - 4(0) - 3$

$y = 0 + 0 - 3$

$y = -3$

The $y$-intercept is $0, -3$.

**Step 4** The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same $y$-value.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3$</td>
<td>$-9$</td>
</tr>
<tr>
<td>$-2$</td>
<td>$-3$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$0$</td>
<td>$-3$</td>
</tr>
<tr>
<td>$1$</td>
<td>$-9$</td>
</tr>
</tbody>
</table>

**Step 5** Connect the points with a smooth curve.
54. \( y = -2x^2 - 8x + 2 \)

**SOLUTION:**

**Step 1** Find the equation of the axis of symmetry. For \( y = -2x^2 - 8x + 2 \), \( a = -2 \), \( b = -8 \), and \( c = 2 \).

\[
x = \frac{-b}{2a}
\]

\[
x = \frac{-( -8)}{2 \cdot ( -2)}
\]

\[
x = \frac{8}{-4}
\]

\[
x = -2
\]

**Step 2** Find the vertex, and determine whether it is a maximum or minimum.

The \( x \)-coordinate of the vertex is \( x = -2 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
f(x) = -2x^2 - 8x + 2
\]

\[
f(-2) = -2(-2)^2 - 8(-2) + 2
\]

\[
f(-2) = -8 + 16 + 2
\]

\[
f(-2) = 10
\]

The vertex lies at \((-2, 10)\). Because \( a \) is negative, the graph opens down, and the vertex is a maximum.

**Step 3** Find the \( y \)-intercept.

Use the original equation, and substitute 0 for \( x \).

\[
y = -2x^2 - 8x + 2
\]

\[
y = -2(0)^2 - 4(0) + 2
\]

\[
y = 0 + 0 + 2
\]

\[
y = 2
\]

The \( y \)-intercept is \((0, 2)\).

**Step 4** The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same \( y \)-value.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>2</td>
<td>8</td>
<td>10</td>
<td>8</td>
<td>2</td>
</tr>
</tbody>
</table>

**Step 5** Connect the points with a smooth curve.
55. \( y = x^2 + 6x - 6 \)

**SOLUTION:**

**Step 1** Find the equation of the axis of symmetry. For \( y = x^2 + 6x - 6 \), \( a = 1 \), \( b = 6 \), and \( c = -6 \).

\[ x = \frac{-b}{2a} \]

\[ x = \frac{-6}{2 \cdot 1} \]

\[ x = -3 \]

**Step 2** Find the vertex, and determine whether it is a maximum or minimum. The \( x \)-coordinate of the vertex is \( x = -3 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[ f(x) = x^2 + 6x - 6 \]

\[ f(-3) = (-3)^2 + 6(-3) - 6 \]

\[ f(-3) = 9 - 18 - 6 \]

\[ f(-3) = -15 \]

The vertex lies at \((-3, -15)\). Because \( a \) is positive, the graph opens up, and the vertex is a minimum.

**Step 3** Find the \( y \)-intercept.

Use the original equation, and substitute 0 for \( x \).

\[ y = x^2 + 6x - 6 \]

\[ y = (0)^2 + 6(0) - 6 \]

\[ y = 0 + 0 - 6 \]

\[ y = -6 \]

The \( y \)-intercept is \((0, -6)\).

**Step 4** The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same \( y \)-value.

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-5)</th>
<th>(-4)</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>(-11)</td>
<td>(-14)</td>
<td>(-15)</td>
<td>(-14)</td>
<td>(-11)</td>
</tr>
</tbody>
</table>

**Step 5** Connect the points with a smooth curve.
56. \( y = x^2 - 2x + 2 \)

SOLUTION:

Step 1 Find the equation of the axis of symmetry. For \( y = x^2 - 2x + 2 \), \( a = 1 \), \( b = -2 \), and \( c = 2 \).

\[
x = \frac{-b}{2a} = \frac{-(-2)}{2 \cdot 1} = \frac{2}{2} = 1
\]

Step 2 Find the vertex, and determine whether it is a maximum or minimum.

The \( x \)-coordinate of the vertex is \( x = 1 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
f(x) = x^2 - 2x + 2
\]

\[
f(1) = (1)^2 - 2(1) + 2 = 1 - 2 + 2 = 1
\]

The vertex lies at \((1, 1)\). Because \( a \) is positive, the graph opens up, and the vertex is a minimum.

Step 3 Find the \( y \)-intercept.

Use the original equation, and substitute 0 for \( x \).

\[
y = x^2 - 2x + 2
\]

\[
y = 0^2 - 2(0) + 2 = 0 + 0 + 2 = 2
\]

The \( y \)-intercept is \((0, 2)\).

Step 4 The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same \( y \)-value.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

Step 5 Connect the points with a smooth curve.
57. \( y = 3x^2 - 12x + 5 \)

**SOLUTION:**

**Step 1** Find the equation of the axis of symmetry. For \( y = 3x^2 - 12x + 5 \), \( a = 3 \), \( b = -12 \), and \( c = 5 \).

\[
\begin{align*}
x &= \frac{-b}{2a} \\
x &= \frac{-(-12)}{2 \cdot (3)} \\
x &= \frac{12}{6} \\
x &= 2
\end{align*}
\]

**Step 2** Find the vertex, and determine whether it is a maximum or minimum. The \( x \)-coordinate of the vertex is \( x = 2 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).

\[
\begin{align*}
f(x) &= 3x^2 - 12x + 5 \\
f(2) &= 3(2)^2 - 12(2) + 5 \\
f(2) &= 12 - 24 + 5 \\
f(2) &= -7
\end{align*}
\]

The vertex lies at \((2, -7)\). Because \( a \) is positive, the graph opens up, and the vertex is a minimum.

**Step 3** Find the \( y \)-intercept.

Use the original equation, and substitute 0 for \( x \).

\[
\begin{align*}
y &= 3x^2 - 12x + 5 \\
y &= 3(0)^2 - 12(0) + 5 \\
y &= 0 + 0 + 5 \\
y &= 5
\end{align*}
\]

The \( y \)-intercept is \((0, 5)\).

**Step 4** The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same \( y \)-value.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5</td>
<td>-4</td>
<td>-7</td>
<td>-4</td>
<td>5</td>
</tr>
</tbody>
</table>

**Step 5** Connect the points with a smooth curve.
58. **BOATING** Miranda has her boat docked on the west side of Casper Point. She is boating over to the Casper Marina. The distance traveled by Miranda over time can be modeled by the equation \( d = -16t^2 + 66t \), where \( d \) is the number of feet she travels in \( t \) minutes.

![Diagram of a boat's path](image)

a. Graph this equation.
b. What is the maximum number of feet north that she traveled?
c. How long did it take her to reach Casper Marina?

**SOLUTION:**

**Step 1** Find the equation of the axis of symmetry. \( d = -16t^2 + 66t \), \( a = -16 \) and \( b = 66 \).

\[
t = \frac{-b}{2a} = \frac{-66}{2 \cdot (-16)} = \frac{66}{32} = \frac{33}{16}
\]

\( t \approx 2 \).

**Step 2** Find the vertex, and determine whether it is a maximum or minimum.
The \( t \)-coordinate of the vertex is \( t = 2 \). Substitute the \( t \)-coordinate of the vertex into the original equation to find the value of \( d \).

\[
d = -16(2)^2 + 66(2) = -64 + 132 = 68
\]

The vertex lies at about \((2, 68)\). Because \( a \) is negative, the graph opens down, and the vertex is a maximum.

**Step 3** Find the \( d \)-intercept.
Use the original equation, and substitute 0 for \( t \).

\[
d = -16t^2 + 66t = -16(0)^2 + 66(0) = 0
\]

The \( d \)-intercept is \((0, 0)\).
9-1 Graphing Quadratic Functions

Step 4 The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same $d$-value.

<table>
<thead>
<tr>
<th>$t$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>50</td>
<td>68</td>
<td>50</td>
</tr>
</tbody>
</table>

Step 5 Connect the points with a smooth curve.

b. The maximum distance Miranda traveled north is the $y$-coordinate of the vertex, or about 68 feet.
c. It takes the boat 2 minutes to reach the vertex, and due to symmetry it will take the boat another 2 minutes to reach Casper Marina. So, it will take Miranda 2 + 2 or about 4 minutes to reach Casper Marina.

GRAPHING CALCULATOR Graph each equation. Use the TRACE feature to find the vertex on the graph. Round to the nearest thousandth if necessary.

59. $y = 4x^2 + 10x + 6$

SOLUTION:
Let $= 4x^2 + 10x + 6$. Use the WINDOW option to adjust the viewing window. Use the TRACE option to move to cursor to the minimum point.

The vertex is at $(-1.25, -0.25)$. 
9-1 Graphing Quadratic Functions

60. \( y = 8x^2 - 8x + 8 \)

**SOLUTION:**

Let \( Y1 = 8x^2 - 8x + 8 \). Use the WINDOW option to adjust the viewing window. Select the TRACE option and move the cursor to the minimum point.

The vertex is at (0.5, 6).

61. \( y = -5x^2 - 3x - 8 \)

**SOLUTION:**

Let \( Y1 = -5x^2 - 3x - 8 \) Use the WINDOW option to adjust the viewing window. Select the TRACE option and move the cursor to the maximum point.

The vertex is at (−0.3,−7.55).

62. \( y = -7x^2 + 12x - 10 \)

**SOLUTION:**

Let \( Y1 = -7x^2 + 12x - 10 \). Use the WINDOW option to adjust the viewing window. Select the TRACE option and move the cursor to the maximum point.

The vertex is at (0.857, −4.857).

63. **GOLF** The average amateur golfer can hit the ball with an initial upward velocity of 31.3 meters per second. The height can be modeled by the equation \( h = -4.9t^2 + 31.3t \), where \( h \) is the height of the ball, in feet, after \( t \) seconds.

a. Graph this equation.
9-1 Graphing Quadratic Functions

b. At what height is the ball hit?
c. What is the maximum height of the ball?
d. How long did it take for the ball to hit the ground?
e. State a reasonable range and domain for this situation.

**SOLUTION:**

**a. Step 1** Find the equation of the axis of symmetry. 
\[ h = -4.9t^2 + 31.3t \]
\[ a = -4.9 \text{ and } b = 31.3. \]

\[ t = \frac{-b}{2a} \]
\[ t = \frac{-(31.3)}{2 \cdot (-4.9)} \]
\[ t = \frac{-31.3}{-9.8} \]
\[ t \approx 3.2 \]

**Step 2** Find the vertex, and determine whether it is a maximum or minimum.
The \( t \)-coordinate of the vertex is \( t = 3.2 \). Substitute the \( t \)-coordinate of the vertex into the original equation to find the value of \( h \).
\[ h = -4.9t^2 + 31.3t \]
\[ h = -4.9(3.2)^2 + 31.3(3.2) \]
\[ h = -50.2 + 100.2 \]
\[ h \approx 50 \]
The vertex lies at about (3.2, 50). Because \( a \) is negative, the graph opens down, and the vertex is a maximum.

**Step 3** Find the \( h \)-intercept.
Use the original equation, and substitute 0 for \( t \).
\[ h = -4.9t^2 + 31.3t \]
\[ h = -4.9(0)^2 + 31.3(0) \]
\[ h = 0 + 0 \]
\[ h = 0 \]
The \( h \)-intercept is (0, 0).

**Step 4** The axis of symmetry divides the parabola into two equal parts. So if there is a point on one side, there is a corresponding point on the other side that is the same distance from the axis of symmetry and has the same \( h \)-value.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3.2</td>
<td>50</td>
</tr>
<tr>
<td>6.4</td>
<td>0</td>
</tr>
</tbody>
</table>

**Step 5** Connect the points with a smooth curve.
**9-1 Graphing Quadratic Functions**

b. The ball is hit when the time is zero, or at the \( t \)-intercept. Since the \( t \)-intercept is \((0, 0)\), the ball is hit at 0 meters.
c. The maximum height of the ball is at the vertex. The vertex is \((3.2, 50)\), so the maximum height of the ball is 50 meters.
d. It takes the ball about 3.2 seconds to reach the vertex, and another 3.2 seconds to come down. Therefore, it takes about 3.2 + 3.2 or about 6.4 seconds to reach the ground.
e. Since the time is zero when the ball is hit and 6.4 when it reaches the ground, the domain is \( D = \{t|0 \leq t \leq 6.4\} \). The ball starts at 0 meters and reaches a maximum height of about 50 meters, so the \( R = \{h|0 \leq h \leq 50.0\} \).

64. **FUNDRAISING** The marching band is selling poinsettias to buy new uniforms. Last year the band charged $5 each, and they sold 150. They want to increase the price this year, and they expect to lose 10 sales for each $1 increase. The sales revenue \( R \), in dollars, generated by selling the poinsettias is predicted by the function \( R = (5 + p)(150 - 10p) \), where \( p \) is the number of $1 price increases.
a. Write the function in standard form.
b. Find the maximum value of the function.
c. At what price should the poinsettias be sold to generate the most sales revenue? Explain your reasoning.

**SOLUTION:**

a. 
\[
R = (5 + p)(150 - 10p) \\
R = (5)(150) - (5)(10p) + (p)(150) - (p)(10p) \\
R = 750 - 50p + 150p - 10p^2 \\
R = -10p^2 + 100p + 750 \\
\]
b. The maximum value of the function occurs at the vertex. In the function \( R = -10p^2 + 100p + 750 \), \( a = -10 \), \( b = 100 \), and \( c = 750 \).
The \( x \)-coordinate of the vertex is \( p = \frac{-b}{2a} \).
\[
p = \frac{100}{2(-10)} \\
p = \frac{-100}{-20} \\
p = 5 \\
\]
The \( p \)-coordinate of the vertex is \( p = 5 \). Substitute the \( p \)-coordinate of the vertex into the original equation to find the value of \( R \).
\[
R = -10p^2 + 100p + 750 \\
R = -10(5)^2 + 100(5) + 750 \\
R = -250 + 500 + 750 \\
R = 1000 \\
The maximum value of the function is 1000.
c. The maximum revenue of $1000, is generated when \( p = 5 \), so by five $1 increases. Since the original price was $5, the new price should be 5 + 5 or $10.
65. **FOOTBALL** A football is kicked up from ground level at an initial upward velocity of 90 feet per second. The equation \( h = -16t^2 + 90t \) gives the height \( h \) of the football after \( t \) seconds.

- **a.** What is the height of the ball after one second?
- **b.** When is the ball 126 feet high?
- **c.** When is the height of the ball 0 feet? What do these points represent in the context of the situation?

**SOLUTION:**

**a.** Substitute \( t = 1 \) into the equation \( h = -16t^2 + 90t \).

\[
h = -16(1)^2 + 90(1) \\
= -16 + 90 \\
= 74
\]

The height of the football after one second is 74 feet.

**b.** Substitute \( h = 126 \) into the equation \( h = -16t^2 + 90t \) and then factor.

\[
h = -16t^2 + 90t \\
126 = -16t^2 + 90t \\
0 = -16t^2 + 90t - 126 \\
0 = -2(8t^2 - 45t + 63) \\
0 = -2(8t^2 - 24t - 21t + 63) \\
0 = -2[8(t - 3) - 21(t - 3)] \\
0 = -2(8t - 21)(t - 3)
\]

This equation is now in the form \( ab = 0 \). If \( 8t - 21 = 0 \) or \( t - 3 = 0 \), then \( t = \frac{21}{8} \) or 2.625 seconds and on the way down at \( t = 3 \) seconds.

**c.** Substitute \( h = 0 \) into the equation \( h = -16t^2 + 90t \) and then factor.

\[
h = -16t^2 + 90t \\
0 = -16t^2 + 90t \\
0 = -2t(8t - 45)
\]

Solve for \( t \).

\[
-2t = 0 \quad \text{or} \quad 8t - 45 = 0 \\
\begin{align*}
t & = 0 \\
8t & = 45 \\
& t = 5.625
\end{align*}
\]

The height of the ball is 0 feet when \( t = 0 \) or \( t = 5.625 \). The height of the ball is 0 feet before the ball is kicked, or when \( t = 0 \) seconds, and it is 0 again when the ball hits the ground after the kick, or when \( t = 5.625 \) seconds.
66. CCSS STRUCTURE Let \( f(x) = x^2 - 9 \).
- a. What is the domain of \( f(x) \)?
- b. What is the range of \( f(x) \)?
- c. For what values of \( x \) is \( f(x) \) negative?
- d. When \( x \) is a real number, what are the domain and range of \( f(x) = \sqrt{x^2 - 9} \)?

**SOLUTION:**
- a. Since \( f(x) \) is a parabola, the domain is all real numbers.
- b. For the function \( f(x) = x^2 - 9 \), \( a = 1 \), \( b = 0 \), and \( c = -9 \). Since \( a \) is positive, this function opens upward so it has a minimum. The range is all real numbers that are greater than or equal to the minimum. The minimum occurs at the vertex.
  
  \[ x = \frac{-b}{2a} \]
  
  \[ x = \frac{0}{2} = 0 \]

  The \( x \)-coordinate of the vertex is \( x = 0 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).
  
  \[ f(x) = x^2 - 9 \]
  
  \[ f(0) = (0)^2 - 9 = -9 \]

  The minimum value is \( -9 \). So the range is all real numbers such that \( y \) is greater than or equal to \( -9 \), \{\( y \) \( \geq -9 \}\}.
  
  - c. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>7</td>
<td>0</td>
<td>-3</td>
<td>-8</td>
<td>-9</td>
<td>-8</td>
<td>-5</td>
<td>0</td>
<td>7</td>
</tr>
</tbody>
</table>

  The function is negative when \( x \) is between \(-3 \) and \( 3 \), so \{\( |x| < 3 \)\}.

  - d. The function is a positive square root. Therefore, the radicand cannot be negative. So,
    
    \[ x^2 - 9 \geq 0 \]
    
    \[ x^2 \geq 9 \]
    
    \[ x \geq 3 \text{ or } x \leq -3 \]

  So, the domain is \{\( x | x \leq -3 \text{ or } x \geq 3 \}\}. Since the function is a square root, the range must be all numbers greater than or equal to zero; \{\( y | y \geq 0 \}\}.

67. MULTIPLE REPRESENTATIONS In this problem, you will investigate solving quadratic equations using tables.

**a. ALGEBRAIC** Determine the related function for each equation. Copy and complete the first two columns of the table below.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Related Function</th>
<th>Zeros</th>
<th>( y )-Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 - x = 12 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x^2 + 8x = 9 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x^2 = 14x - 24 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x^2 + 16x = -28 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
b. **GRAPHICAL** Graph each related function with a graphing calculator.

c. **ANALYTICAL** Use the table feature on your calculator to determine the zeros of each related function. Record the zeros in the table above. Also record the values of the function one unit less than and one unit more than each zero.

d. **VERBAL** Examine the function values for \( x \)-values just before and just after a zero. What happens to the sign of the function value before and after a zero?

**SOLUTION:**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Related Function</th>
<th>Zeros</th>
<th>( y )-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 - x = 12 )</td>
<td>( y = x^2 - x - 12 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x^2 + 8x = 9 )</td>
<td>( y = x^2 + 8x - 9 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x^2 = 14x - 24 )</td>
<td>( y = x^2 - 14x + 24 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x^2 + 16x = -28 )</td>
<td>( y = x^2 + 16x + 28 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b.

Let \( Y_1 = x^2 - x - 12 \). Select 6 Under the **ZOOM** option to adjust the viewing window to standard.

Let \( Y_1 = x^2 + 8x - 9 \). Use the **WINDOW** option to adjust the viewing window.

Let \( Y_1 = x^2 - 14x + 24 \). Use the **WINDOW** option to adjust the viewing window.
9-1 Graphing Quadratic Functions

Let \( Y_1 = x^2 + 16x + 28 \). Use the WINDOW option to adjust the viewing window.

\[
\begin{array}{c|c|c}
\hline
X & Y_1 & X \\
\hline
-4 & 8 & -1 \\
-3 & 10 & 0 \\
-2 & 12 & 1 \\
-1 & 10 & 2 \\
0 & 8 & 3 \\
1 & 6 & 4 \\
2 & 4 & 5 \\
3 & 2 & 6 \\
4 & 0 & 7 \\
5 & 8 & 8 \\
\hline
\end{array}
\]

Let \( Y_1 = x^2 - x - 12 \). Use the 2nd [TABLE] option to display a table of values. Use the up and down arrow to locate the lines where \( Y_1 = 0 \). Identify the \( y \)-values above and below the zero.

\[
\begin{array}{c|c|c}
\hline
X & Y_1 & X \\
\hline
-10 & 10 & -1 \\
-9 & 12 & 0 \\
-8 & 14 & 1 \\
-7 & 16 & 2 \\
-6 & 18 & 3 \\
-5 & 20 & 4 \\
-4 & 22 & 5 \\
-3 & 24 & 6 \\
-2 & 26 & 7 \\
-1 & 28 & 8 \\
0 & 30 & 9 \\
1 & 32 & 10 \\
2 & 34 & 11 \\
3 & 36 & 12 \\
\hline
\end{array}
\]

Let \( Y_1 = x^2 + 8x - 9 \). Use the 2nd [TABLE] option to display a table of values. Use the up and down arrow to locate the lines where \( Y_1 = 0 \). Identify the \( y \)-values above and below the zero.

\[
\begin{array}{c|c|c}
\hline
X & Y_1 & X \\
\hline
-10 & 11 & -1 \\
-9 & 9 & 0 \\
-8 & 7 & 1 \\
-7 & 5 & 2 \\
-6 & 3 & 3 \\
-5 & 1 & 4 \\
-4 & -1 & 5 \\
-3 & -3 & 6 \\
-2 & -5 & 7 \\
-1 & -7 & 8 \\
0 & -9 & 9 \\
1 & -11 & 10 \\
2 & -13 & 11 \\
3 & -15 & 12 \\
\hline
\end{array}
\]

Let \( Y_1 = x^2 - 14x + 24 \). Use the 2nd [TABLE] option to display a table of values. Use the up and down arrow to
9-1 Graphing Quadratic Functions

locate the lines where \( Y_1 = 0 \). Identify the \( y \)-values above and below the zero.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-9</td>
</tr>
<tr>
<td>-16</td>
<td>-21</td>
</tr>
<tr>
<td>-24</td>
<td>-11</td>
</tr>
<tr>
<td>0</td>
<td>11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-9</td>
</tr>
<tr>
<td>-16</td>
<td>-21</td>
</tr>
<tr>
<td>-24</td>
<td>-11</td>
</tr>
<tr>
<td>0</td>
<td>11</td>
</tr>
</tbody>
</table>

Let \( Y_1 = x^2 + 16x + 28 \). Use the 2nd [TABLE] option to display a table of values. Use the up and down arrow to locate the lines where \( Y_1 = 0 \). Identify the \( y \)-values above and below the zero.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-13</td>
<td>0</td>
</tr>
<tr>
<td>-12</td>
<td>-11</td>
</tr>
<tr>
<td>-9</td>
<td>-20</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-13</td>
<td>0</td>
</tr>
<tr>
<td>-12</td>
<td>-11</td>
</tr>
<tr>
<td>-9</td>
<td>-20</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>13</td>
</tr>
</tbody>
</table>

\begin{array}{|c|c|c|c|}
\hline
\text{Equation} & \text{Related Function} & \text{Zeros} & \text{\( y \)-values} \\
\hline
x^2 - x = 12 & y = x^2 - x - 12 & -3, 4 & -3: 8, -6; 4: -6, 8 \\
\hline
x^2 + 8x = 9 & y = x^2 - 8x + 9 & -9, 1 & -9: 11, -9; 1: -9, 11 \\
\hline
x^2 = 14x - 24 & y = x^2 - 14x + 24 & 2, 12 & 2: 11, -9; 12: -9, 11 \\
\hline
x^2 + 16x = -28 & y = x^2 + 16x + 28 & -14, -2 & -14: 13, -11: -2: -11, 13 \\
\hline
\end{array}

\textbf{d.} The function values have opposite signs before and just after a zero.

\textbf{68. OPEN ENDED} Write a quadratic function for which the graph has an axis of symmetry of \( x = -\frac{3}{8} \). Summarize your steps.

\textbf{SOLUTION:}

The axis of symmetry is the line \( x = -\frac{b}{2a} \). Since we want this line to be \( x = -\frac{3}{8} \), let \( b = 3 \) and let \( 2a = 8 \), or \( a = 4 \).

Write a quadratic function by substituting 4 for \( a \), 3 for \( b \), and any value for \( c \).

\[ y = ax^2 + bx + c \quad \text{General form of quadratic function} \]

\[ y = 4x^2 + 3x + 5 \quad a = 4, b = 3, c = 5 \]

A quadratic function for which the graph has an axis of symmetry of \( x = -\frac{3}{8} \) could be \( y = 4x^2 + 3x + 5 \).
9-1 Graphing Quadratic Functions

69. **ERROR ANALYSIS** Jade thinks that the parabolas represented by the graph and the description have the same axis of symmetry. Chase disagrees. Who is correct? Explain your reasoning.

![Graph](image)

**SOLUTION:**

Determine the axis of symmetry for the description and the graph.

**Description:**

The parabola opens downward, so it is a vertical graph. The vertex is at (2, 2). The axis of symmetry for a vertical graph (graphs of the form \( y = x^2 \)), is always a vertical line that passes through the \( x \)-coordinate of the vertex. Therefore, the axis of symmetry is \( x = 2 \).

**Graph:**

The vertex of the graph appears to be located at the \( x \)-coordinate of about 1.5. This tells us that the axis of symmetry will be \( x = 1.5 \).

Chase; the lines of symmetry are \( x = 1.5 \) and \( x = 2 \).
9-1 Graphing Quadratic Functions

70. CHALLENGE Using the axis of symmetry, the y-intercept, and one x-intercept, write an equation for the graph shown.

SOLUTION:
The graph is a parabola with a vertex at (3, 25). So, the axis of symmetry is the line $x = 3$. Substitute 3 in for $x$ in the formula for axis of symmetry and solve for $b$ in terms of $a$.

$$x = \frac{-b}{2a}$$  \hspace{1cm} \text{Equation of axis of symmetry}

$$3 = \frac{-b}{2a}$$  \hspace{1cm} \text{Axis of symmetry is 3.}

$$3 \cdot (-2a) = \frac{-b}{2a} \cdot (-2a)$$  \hspace{1cm} \text{Multiply each side by } -2a.

$$-6a = b$$  \hspace{1cm} \text{Simplify}

The parabola has a y-intercept of 16 and x-intercepts at (-2, 0) and (8, 0).

The standard form of the equation for a parabola is $y = ax^2 + bx + c$. Substitute $-6a$ for $b$, 16 for $c$, and (8, 0) for $(x, y)$ to determine the equation for this graph.

$$y = ax^2 + bx + c$$  \hspace{1cm} \text{Standard form of quadratic equation}

$$y = ax^2 - 6ax + 16$$

$$0 = a(8)^2 - 6(8)x + 16$$  \hspace{1cm} (x, y) = (8, 0).

$$0 = 64a - 48a + 16$$  \hspace{1cm} \text{Simplify.}

$$0 = 16a + 16$$

$$-16a = 16$$  \hspace{1cm} \text{Subtract 16a from each side.}

$$a = -1$$  \hspace{1cm} \text{Divide each side by } -16.

The value of $a = -1$, so the value of $b$ is $-6(-1)$ or 6.

Therefore, the equation for the graph shown is $y = -x^2 + 6x + 16$. 
9-1 Graphing Quadratic Functions

71. **CCSS STRUCTURE**  The graph of a quadratic function has a vertex at (2, 0). One point on the graph is (5, 9). Find another point on the graph. Explain how you found it.

**SOLUTION:**
I graphed the points given, and the parabola formed by them. Since a parabola is symmetric around the vertex, I counted the spaces right and up from the vertex to (5, 9). I had to move 3 spaces right, and 9 spaces up. Due to the symmetry, I know that I can move 3 spaces to the left and 9 spaces up from (2, 0) to find another point. By doing so, I arrive at the point (−1, 9).

![Graph of a parabola with points (−1, 9) and (5, 9)]

72. **OPEN ENDED**  Describe a real-world situation that involves a quadratic equation. Explain what the vertex represents.

**SOLUTION:**
The path travelled by a football kicked during a game is described by a quadratic. The vertex gives the maximum height of the flight of the ball.

73. **REASONING**  Provide a counterexample to the following statement. *The vertex of a parabola is always the minimum of the graph.*

**SOLUTION:**
The function \( y = -x^2 - 4 \) has a vertex at (0, −4), but because \( a \) is negative and the parabola opens down, this point is a maximum.

![Graph of a parabola with vertex at (0, -4)]

74. **WRITING IN MATH**  Use tables and graphs to compare and contrast an exponential function \( f(x) = ab^x + c \), where \( a \neq 0, b > 0, \) and \( b \neq 1 \), a quadratic function \( g(x) = ax^2 + c \), and a linear function \( h(x) = ax + c \). Include intercepts, portions of the graph where the functions are increasing, decreasing, positive, or negative, relative maxima and minima, symmetries, and end behavior. Which function eventually exceeds the others?

**SOLUTION:**
Use basic values for each variable.

Sample answer: Suppose \( a = 1, b = 2, \) and \( c = 1 \).
9-1 Graphing Quadratic Functions

<table>
<thead>
<tr>
<th>x</th>
<th>f(x) = 2^x + 1</th>
<th>g(x) = x^2 + 1</th>
<th>h(x) = x + 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>1.00098</td>
<td>101</td>
<td>-9</td>
</tr>
<tr>
<td>-8</td>
<td>1.00391</td>
<td>65</td>
<td>-7</td>
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<tr>
<td>-6</td>
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<td>-4</td>
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<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>5</td>
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<td>17</td>
<td>5</td>
</tr>
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<td>65</td>
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<td>8</td>
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</tr>
<tr>
<td>10</td>
<td>1205</td>
<td>101</td>
<td>11</td>
</tr>
</tbody>
</table>

Intercepts: f(x) and g(x) have no x-intercepts, h(x) has one at -1 because c = 1. g(x) and h(x) have one y-intercept at 1 and f(x) has one y-intercept at 2. The graphs are all shifted up 1 unit from the graph of the parent functions because c = 1.

Increasing/Decreasing: f(x) and h(x) are increasing on the entire domain. g(x) is increasing to the right of the vertex and decreasing to the left.

Positive/Negative: The function values for f(x) and g(x) are all positive. The function values of h(x) are negative for x < -1 and positive for x > -1.

Maxima/Minima: f(x) and h(x) have no maxima or minima. g(x) has a minimum at (0, 1).

Symmetry: f(x) and g(x) have no symmetry. g(x) is symmetric about the y-axis.

End behavior: For f(x) and h(x), as x increases, y increases and as x decreases, y decreases. For g(x), as x increases, y increases and as x decreases, y increases. The exponential function f(x) eventually exceeds the others.
9-1 Graphing Quadratic Functions

75. Which of the following is an equation for the line that passes through (2, −5) and is perpendicular to 2x + 4y = 8?

A $y = 2x + 10$
B $y = -\frac{1}{2}x - 4$
C $y = 2x - 9$
D $y = -2x - 1$

**SOLUTION:**
First, rewrite the equation of the line in slope-intercept form.
$2x + 4y = 8$
$4y = -2x + 8$
$y = -\frac{1}{2}x + 2$

The slope of the line with equation $y = -\frac{1}{2}x + 2$ is $-\frac{1}{2}$. The slope of the perpendicular line is the opposite reciprocal of $-\frac{1}{2}$, or 2. Use the slope and the given point to find the y-intercept.

$y = mx + b$
$-5 = 2(2) + b$
$-5 = 4 + b$
$-9 = b$

Write the equation in the form of $y = mx + b$: $y = 2x - 9$. So, The correct choice is C.

76. **GEOMETRY**  The area of the circle is $36\pi$ square units. If the radius is doubled, what is the area of the new circle?

F $72\pi$ units$^2$
G $144\pi$ units$^2$
H $1296\pi$ units$^2$
J $9\pi$ units$^2$

**SOLUTION:**
The area of a circle is given by $A = \pi r^2$.
$A = 36\pi$
$= (6)^2 \pi$

Therefore, the radius of the circle given is 6 units. If the radius is doubled, the new radius is $2 \cdot 6$ or 12 units.
$A = \pi r^2$
$= \pi (12)^2$
$= 144\pi$

The area of the new circle is $144\pi$ units$^2$. So, The correct choice is G.
9-1 Graphing Quadratic Functions

77. What is the range of the function \( f(x) = -4x^2 - \frac{1}{2} \)?

\[ \text{A} \{ \text{all integers less than or equal to } \frac{1}{2} \} \]
\[ \text{B} \{ \text{all nonnegative integers} \} \]
\[ \text{C} \{ \text{all real numbers} \} \]
\[ \text{D} \{ \text{all real numbers less than or equal to } -\frac{1}{2} \} \]

\textbf{SOLUTION:} \\
The function \( f(x) = -4x^2 - \frac{1}{2} \) is a parabola, with \( a = -4, b = 0, \) and \( c = -\frac{1}{2} \). Because \( a \) is negative, the graph opens downward, and the function has a maximum value. The range is all real numbers less than or equal to the maximum value. \\
The maximum value is at the vertex. \\
The \( x \)-coordinate of the vertex is \( x = -\frac{b}{2a} \).
\[
x = \frac{-b}{2a} = \frac{-0}{2 \cdot (-4)} = 0
\]

The \( x \)-coordinate of the vertex is \( x = 0 \). Substitute the \( x \)-coordinate of the vertex into the original equation to find the value of \( y \).
\[
f(x) = -4x^2 - \frac{1}{2}
\]
\[
f(0) = -4(0)^2 - \frac{1}{2} = -\frac{1}{2}
\]

The vertex is at \( \left( 0, -\frac{1}{2} \right) \).

The maximum value is \( -\frac{1}{2} \). \\
The range is all real numbers less than or equal to the maximum value, or all real numbers less than or equal to \( -\frac{1}{2} \). \\
So, the correct choice is \( \text{D} \).

78. \textbf{SHORT RESPONSE} Dylan delivers newspapers for extra money. He starts delivering the newspapers at 3:15 P.M. and finishes at 5:05 P.M. How long does it take Dylan to complete his route?

\textbf{SOLUTION:} \\
Dylan starts delivering the newspapers at 3:15. From 3:15 to 4:00, it is 45 minutes. From 4:00 to 5:00, it is 60 minutes. Finally, from 5:00 to 5:05 it is 5 minutes. \\
So, it takes Dylan 45 + 60 + 5 or 110 minutes, which is 1 hour and 50 minutes to complete his route.
Determine whether each trinomial is a perfect square trinomial. Write yes or no. If so, factor it.

79. $4x^2 + 4x + 1$

**SOLUTION:**
The first term is a perfect square.
$4x^2 = (2x)^2$
The last term is a perfect square.
$1 = (1)^2$
The middle term is equal to $2ab$.
$4x = 2(2x)(1)$
So, $4x^2 + 4x + 1$ is a perfect square trinomial.
$4x^2 + 4x + 1 = (2x)^2 + 2(2x)(1) + (1)^2$
$= (2x + 1)^2$

80. $4x^2 - 20x + 25$

**SOLUTION:**
The first term is a perfect square.
$4x^2 = (2x)^2$
The last term is a perfect square.
$25 = (5)^2$
The middle term is equal to $2ab$.
$20x = 2(2x)(5)$
So, $4x^2 - 20x + 25$ is a perfect square trinomial.
$4x^2 - 20x + 25 = (2x)^2 - 2(2x)(5) + (5)^2$
$= (2x - 5)^2$

81. $9x^2 + 8x + 16$

**SOLUTION:**
The first term is a perfect square.
$9x^2 = (3x)^2$
The last term is a perfect square.
$16 = (4)^2$
The middle term is not equal to $2ab$.
$8x \neq 2(3x)(4)$
So, $9x^2 + 8x + 16$ is not a perfect square trinomial.

**Factor each polynomial if possible. If the polynomial cannot be factored, write prime.**

82. $n^2 - 16$

**SOLUTION:**

$n^2 - 16 = n^2 - 4^2$
$= (n - 4)(n + 4)$
9-1 Graphing Quadratic Functions

83. \( x^2 + 25 \)

**SOLUTION:**
This polynomial cannot be factored. It is prime.

84. \( 9 - 4a^2 \)

**SOLUTION:**
\[ 9 - 4a^2 = 3^2 - (2a)^2 = (3 - 2a)(3 + 2a) \]

Find each product.
85. \((b - 7)(b + 3)\)

**SOLUTION:**
\[ (b - 7)(b + 3) = b^2 + 3b - 7b - 21 = b^2 - 4b - 21 \]

86. \((c - 6)(c - 5)\)

**SOLUTION:**
\[ (c - 6)(c - 5) = c^2 - 5c - 6c + 30 = c^2 - 11c + 30 \]

87. \((2x - 1)(x + 9)\)

**SOLUTION:**
\[ (2x - 1)(x + 9) = 2x^2 + 18x - x - 9 = 2x^2 + 17x - 9 \]

88. **MULTIPLE BIRTHS** The number of quadruplet births \(Q\) in the United States in recent years can be modeled by \(Q = -0.5r^3 + 11.7r^2 - 21.5r + 218.6\), where \(r\) represents the number of years since 2002. What is the expected number of quadruplet births in the United States in 2017?

**SOLUTION:**
Since \(r\) is the number of years since 2002, \(r\) equals 2017 - 2002 or 15.
\[ Q = -0.5x^3 + 11.7x^2 - 21.5x + 218.6 \quad \text{Original equation} \]
\[ = -0.5(15)^3 + 11.7(15)^2 - 21.5(15) + 218.6 \quad x = 15 \]
\[ = -0.5(3375) + 11.7(225) - 21.5(15) + 218.6 \quad \text{Simplify.} \]
\[ = -1678.5 + 2632.5 - 322.5 + 218.6 \quad \text{Multiply.} \]
\[ = 841.1 \quad \text{Simplify.} \]
So, the number of quadruplet births in the United States in 2017 is expected to be about 841.
9-1 Graphing Quadratic Functions

Use elimination to solve each system of equations.

89. \[2x + y = 5\]
\[3x - 2y = 4\]

**SOLUTION:**
Notice that if you multiply the first equation by 2, the coefficients of the y-terms are additive inverses.

\[2x + y = 5\] (Multiply by 2) \[4x + 2y = 10\]
\[3x - 2y = 4\] \[\text{(+) 3x - 2y = 4}\]
\[7x = 14\]
\[x = 2\]

Now, substitute 2 for \(x\) in either equation to find \(y\).

\[2x + y = 5\]
\[2(2) + y = 5\]
\[4 + y = 5\]
\[y = 1\]

The solution is \((2, 1)\).

90. \[4x - 3y = 12\]
\[x + 2y = 14\]

**SOLUTION:**
Notice that if you multiply the second equation by \(-4\), the coefficients of the \(x\)-terms are additive inverses.

\[4x - 3y = 12\]
\[4x - 3y = 12\]
\[x + 2y = 14\] (Multiply by \(-4\)) \[\text{(+) - 4x - 8y = -56}\]
\[-11y = -44\]
\[y = 4\]

Now, substitute 4 for \(y\) in either equation to find \(x\).

\[x + 2y = 14\]
\[x + 2(4) = 14\]
\[x + 8 = 14\]
\[x = 6\]

The solution is \((6, 4)\).
9.1 Graphing Quadratic Functions

91. \(2x - 3y = 2\)
\(5x + 4y = 28\)

**SOLUTION:**
Notice that if you multiply the first equation by 4 and the second equation by 3, the coefficients of the \(y\)-terms are additive inverses.
\[
\begin{align*}
2x - 3y &= 2 \quad \text{Multiply by 4.} \quad 8x - 12y &= 8 \\
5x + 4y &= 28 \quad \text{Multiply by 3.} \quad (+) \quad 15x + 12y &= 84 \\
\hline
23x &= 92 \\
\therefore x &= 4
\end{align*}
\]
Now, substitute 4 for \(x\) in either equation to find \(y\).
\[
\begin{align*}
5x + 4y &= 28 \\
5(4) + 4y &= 28 \\
20 + 4y &= 28 \\
4y &= 8 \\
\therefore y &= 2
\end{align*}
\]
The solution is (4, 2).

92. **HEALTH** About 20\% of the time you sleep is spent in rapid eye movement (REM), which is associated with dreaming. If an adult sleeps 7 to 8 hours, how much time is spent in REM sleep?

**SOLUTION:**
Twenty percent of 7 hours is \(0.20 \times 7\) or 1.4.
Twenty percent of 8 hours is \(0.20 \times 8\) or 1.6.
Therefore, between 1.4 and 1.6 hours, inclusive, is spent in REM sleep.

**Find the \(x\)-intercept of the graph of each equation.**

93. \(x + 2y = 10\)

**SOLUTION:**
To find the \(x\)-intercept, let \(y = 0\).
\[
\begin{align*}
x + 2y &= 10 \\
x + 2(0) &= 10 \\
x + 0 &= 10 \\
x &= 10
\end{align*}
\]
The \(x\)-intercept is (10, 0).
9-1 Graphing Quadratic Functions

94. $2x - 3y = 12$

**SOLUTION:**
To find the $x$-intercept, let $y = 0$.

$2x - 3y = 12$
$2x - 3(0) = 12$
$2x - 0 = 12$
$2x = 12$
$\frac{2x}{2} = \frac{12}{2}$
$x = 6$

The $x$-intercept is (6, 0).

95. $3x - y = -18$

**SOLUTION:**
To find the $x$-intercept, let $y = 0$.

$3x - y = -18$
$3x - (0) = -18$
$3x = -18$
$\frac{3x}{3} = \frac{-18}{3}$
$x = -6$

The $x$-intercept is (-6, 0).