8-5 Using the Distributive Property

Use the Distributive Property to factor each polynomial.

1. \(21b - 15a\)
   
   SOLUTION:
   
   \[21b = 3 \cdot 7 \cdot b\]
   \[15a = 3 \cdot 5 \cdot a\]
   
The greatest common factor in each term is 3.
   
   \[21b - 15a = 3(7b) + 3(-5a) = 3(7b - 5a)\]

2. \(14c^2 + 2c\)
   
   SOLUTION:
   
   \[14c^2 = 2 \cdot 7 \cdot c \cdot c\]
   \[2c = 2 \cdot 5 \cdot c\]
   
The greatest common factor in each term is \(2c\).
   
   \[14c^2 + 2c = 2c(7c) + 2c = 2c(7c + 1)\]

3. \(10g^2h^2 + 9gh^2 - g^2h\)
   
   SOLUTION:
   
   \[10g^2h^2 = 2 \cdot 5 \cdot g \cdot g \cdot h \cdot h\]
   \[9gh^2 = 3 \cdot 3 \cdot g \cdot g \cdot h\]
   \[g^2h = g \cdot g \cdot h\]
   
The greatest common factor in each term is \(gh\).
   
   \[10g^2h^2 + 9gh^2 - g^2h = gh(10gh) + gh(9h) + gh(-g) = gh(10gh + 9h - g)\]

4. \(12j^2k^2 + 6j^2k + 2j^2k^2\)
   
   SOLUTION:
   
   \[12j^2k^2 = 2 \cdot 2 \cdot 3 \cdot j \cdot k \cdot k\]
   \[6j^2k = 2 \cdot 3 \cdot j \cdot j \cdot k\]
   \[2j^2k^2 = 2 \cdot j \cdot j \cdot k \cdot k\]
   
The greatest common factor in each term is \(2jk\).
   
   \[12j^2k^2 + 6j^2k + 2j^2k^2 = 2jk(6k) + 2jk(3j) + 2jk(jk) = 2jk(6k + 3j + jk)\]
Factor each polynomial.

5. \(np + 2n + 8p + 16\)

**SOLUTION:**

\[
np + 2n + 8p + 16 = (np + 2n) + (8p + 16) \quad \text{Group terms w/ common factors.}
\]

\[
= n(p + 2) + 8(p + 2) \quad \text{Factor the GCF \((p+2)\).}
\]

\[
= (n + 8)(p + 2) \quad \text{Dist. Prop.}
\]

6. \(xy - 7x + 7y - 49\)

**SOLUTION:**

\[
xy - 7x + 7y - 49 = (xy + 7y) + (-7x - 49) \quad \text{Group terms w/ common factors.}
\]

\[
= y(x + 7) + (-7)(x + 7) \quad \text{Factor GCF from each group.}
\]

\[
= (x + 7)(y - 7) \quad \text{Distributive Property}
\]

7. \(3bc - 2b - 10 + 15c\)

**SOLUTION:**

\[
3bc - 2b - 10 + 15c = (3bc + 15c) + (-2b - 10) \quad \text{Group terms with common factors.}
\]

\[
= 3c(b + 5) + (-2)(b + 5) \quad \text{Factor the GCF from each group.}
\]

\[
= (b + 5)(3c - 2) \quad \text{Common term \((b+5)\), Distributive Property}
\]

8. \(9g - 45f - 7g + 35\)

**SOLUTION:**

\[
9g - 45f - 7g + 35 = (9g - 45f) + (-7g + 35) \quad \text{Group terms with common factors.}
\]

\[
= 9g(g - 5) + (-7)(g - 5) \quad \text{Factor the GCF from each group.}
\]

\[
= (9f - 7)(g - 5) \quad \text{Common factor \((g - 5)\), Distributive Property}
\]
8-5 Using the Distributive Property

Solve each equation. Check your solutions.

9. $3k(k + 10) = 0$

**SOLUTION:**
Since 0 is on one side of the equation and the other side is in factor form, apply the Zero Product Property and set each factor equal to 0. Solve each of the resulting equations.

$3k(k + 10) = 0$

$3k = 0$ or $k + 10 = 0$

$k = 0$ or $k = -10$

The roots are 0 and -10. Check by substituting 0 and -10 in for $k$ in the original equation.

$3k(k + 10) = 0$

$3(0)(0 + 10) \neq 0$

$3(-10)(-10 + 10) \neq 0$

$0(10) = 0$

So, the solutions are 0 and -10.

10. $(4m + 2)(3m - 9) = 0$

**SOLUTION:**
Since 0 is on one side of the equation and the other side is in factor form, apply the Zero Product Property and set each factor equal to 0. Solve each of the resulting equations.

$(4m + 2)(3m - 9) = 0$

$4m + 2 = 0$ or $3m - 9 = 0$

$4m = -2$ or $3m = 9$

$m = -\frac{1}{2}$ or $m = 3$

The roots are $-\frac{1}{2}$ and 3. Check by substituting $-\frac{1}{2}$ and 3 in for $m$ in the original equation.

$(4m + 2)(3m - 9) = 0$

$[4\left(-\frac{1}{2}\right) + 2][3\left(-\frac{1}{2}\right) - 9] \neq 0$

$[-2 + 2][-\frac{3}{2} - 9] \neq 0$

$0\left(-\frac{21}{2}\right) \neq 0$

So, the solutions are $-\frac{1}{2}$ and 3.
11. \(20p^2 - 15p = 0\)

**SOLUTION:**
Since 0 is on one side of the equation, factor the other side. Next, apply the Zero Product Property by setting each factor equal to 0. Solve each of the resulting equations.

\[20p^2 - 15p = 0\]
\[5p(4p - 3) = 0\]

\[4p - 3 = 0\]
\[5p = 0 \quad \text{or} \quad 4p = 3\]
\[p = 0 \quad \text{or} \quad p = \frac{3}{4}\]

The roots are 0 and \(\frac{3}{4}\). Check by substituting 0 and \(\frac{3}{4}\) in for \(p\) in the original equation.

\[20p^2 - 15p = 0\]
\[20(0)^2 - 15(0) \neq 0\]
\[20(0) - 0 \neq 0\]
\[0 - 0 \neq 0\]
\[0 = 0 \checkmark\]

So, the solutions are 0 and \(\frac{3}{4}\).

12. \(r^2 = 14r\)

**SOLUTION:**
Rewrite the equation so one side has 0 and the other side is in factor form. Next, apply the Zero Product Property by setting each factor equal to 0. Solve each of the resulting equations.

\[r^2 = 14r\]
\[r^2 - 14r = 0\]
\[r(r - 14) = 0\]

\[r = 0 \quad \text{or} \quad r - 14 = 0\]

\[r = 0 \quad \text{or} \quad r = 14\]

The roots are 0 and 14. Check by substituting 0 and 14 in for \(r\) in the original equation.

\[r^2 = 14r\]
\[(0)^2 \neq 14(0)\]
\[(14)^2 \neq 14(14)\]
\[0 = 0 \checkmark\]

So, the solutions are 0 and 14.
8-5 Using the Distributive Property

13. **SPIDERS** Jumping spiders can commonly be found in homes and barns throughout the United States. A jumping spider’s jump can be modeled by the equation $h = 33.3t - 16t^2$, where $t$ represents the time in seconds and $h$ is the height in feet.
   a. When is the spider’s height at 0 feet?
   b. What is the spider’s height after 1 second? after 2 seconds?

**SOLUTION:**
   a. Write the equation so it is of the form $ab = 0$.
   $33.3t - 16t^2 = 0$
   $t(33.3 - 16t) = 0$
   $t = 0$ or $33.3 = 16t$
   $t = 0$ or $33.3 = 16t$
   $t = 0$ or $2.08125 = t$
   The spider’s height is at 0 ft. at 0 sec. and at 2.08125 sec.
   b. $h = 33.3(1) - 16(1)^2 = 17.3$
   $h = 33.3(2) - 16(2)^2 = 2.6$
   The spider’s height after 1 second is 17.3 ft. and after 2 seconds is 2.6 ft.

14. **CCSS REASONING** At a Fourth of July celebration, a rocket is launched straight up with an initial velocity of 125 feet per second. The height $h$ of the rocket in feet above sea level is modeled by the formula $h = 125t - 16t^2$, where $t$ is the time in seconds after the rocket is launched.
   a. What is the height of the rocket when it returns to the ground?
   b. Let $h = 0$ in the equation and solve for $t$.
   c. How many seconds will it take for the rocket to return to the ground?

**SOLUTION:**
   a. The height of the rocket when it returns to the ground is 0 ft.
   b. $0 = 125t - 16t^2$
   $0 = t(125 - 16t)$
   $125 - 16t = 0$
   $t = 0$ or $125 = 16t$
   $t = 0$ or $7.8125$
   c. It will take the rocket about 7.8 seconds to return to the ground.

**Use the Distributive Property to factor each polynomial.**
15. $16t - 40y$

**SOLUTION:**
   $16t = 2 \cdot 2 \cdot 2 \cdot 2 \cdot t$
   $40y = 2 \cdot 2 \cdot 2 \cdot 5 \cdot y$
   The greatest common factor in each term is 8.
   $16t - 40y = 8(2t) + 8(-5y)$
   $= 8(2t - 5y)$
16. $30v + 50x$

**SOLUTION:**

$30v = 2 \cdot 3 \cdot 5 \cdot v$

$50x = 2 \cdot 5 \cdot 5 \cdot x$

The greatest common factor in each term is 10.

$30v + 50x = 10(3v) + 10(5x)$

$= 10(3v + 5x)$

17. $2k^2 + 4k$

**SOLUTION:**

$2k^2 = 2 \cdot k \cdot k$

$4k = 2 \cdot 2 \cdot k$

The greatest common factor in each term is $2k$.

$2k^2 + 4k = 2k(k) + 2k(2)$

$= 2k(k + 2)$

18. $5x^2 + 10z$

**SOLUTION:**

$5x^2 = 5 \cdot z \cdot z$

$10z = 2 \cdot 5 \cdot z$

The greatest common factor in each term is $5x$.

$5x^2 + 10z = 5x(z) + 5z(2)$

$= 5x(z + 2)$

19. $4a^2b^2 + 2a^2b - 10ab^2$

**SOLUTION:**

$4a^2b^2 = 2 \cdot 2 \cdot a \cdot a \cdot b \cdot b$

$2a^2b = 2 \cdot a \cdot b \cdot b$

$10ab^2 = 2 \cdot 5 \cdot a \cdot b \cdot b$

The greatest common factor in each term is $2ab$.

$4a^2b^2 + 2a^2b - 10ab^2 = 2ab(2ab) + 2ab(a) + 2ab(-5b)$

$= 2ab(2ab + a - 5b)$
8-5 Using the Distributive Property

20. $5c^2v - 15c^2v^2 + 5c^2v^3$

**SOLUTION:**

$5c^2v = 5 \cdot c \cdot c \cdot v$

$15c^2v^2 = 3 \cdot 5 \cdot c \cdot c \cdot v \cdot v$

$5c^2v^3 = 5 \cdot c \cdot c \cdot v \cdot v \cdot v$

The greatest common factor in each term is $5c^2v$.

$5c^2v - 15c^2v^2 + 5c^2v^3$

$= 5c^2v(1) + 5c^2v(-3v) + 5c^2v(v^2)$

$= 5c^2v(1 - 3v + v^2)$

**Factor each polynomial.**

21. $fg - 5g + 4f - 20$

**SOLUTION:**

$f'g - 5g + 4f - 20$

$= (fg - 5g) + (4f - 20)$ Group terms with common factors.

$= g(f - 5) + 4(f - 5)$ Factor the GCF from each group.

$= (g + 4)(f - 5)$ Common factor $(f - 5)$, Distributive Property

22. $a^2 - 4a - 24 + 6a$

**SOLUTION:**

$a^2 - 4a - 24 + 6a$

$= (a^2 - 4a) + (6a - 24)$ Group terms with common factors.

$= a(a - 4) + 6(a - 4)$ Factor GCF from each group.

$= (a - 4)(a + 6)$ Common factor $(a - 4)$, Distributive Property

23. $hj - 2h + 5j - 10$

**SOLUTION:**

$hj - 2h + 5j - 10$

$= (hj - 2h) + (5j - 10)$ Group terms with common factors.

$= h(j - 2) + 5(j - 2)$ Factor GCF from each group.

$= (h + 5)(j - 2)$ Common factor $(j - 2)$, Dist. Prop.
8-5 Using the Distributive Property

24. \( xy - 2x - 2 + y \)

**SOLUTION:**

\[
xy - 2x - 2 + y = (xy - 2x) + (y - 2) \quad \text{Group terms with common factors.}
\]

\[
x(y - 2) + (y - 2) = (x + 1)(y - 2) \quad \text{Factor GCF from each group.}
\]

\[
(\text{Common factor } (y-2), \text{ Dist. Prop.})
\]

25. \( 45pq - 27q - 50p + 30 \)

**SOLUTION:**

\[
45pq - 27q - 50p + 30 = (45pq - 50p) + (-27q + 30) \quad \text{Group terms with common factors.}
\]

\[
5p(9q - 10) + (-3)(8q - 10) \quad \text{Factor GCF from each group.}
\]

\[
(9q - 10)(5p - 3) \quad \text{Common factor } (9q-10), \text{ Dist. Prop.}
\]

26. \( 24ty - 18t + 4y - 3 \)

**SOLUTION:**

\[
24ty - 18t + 4y - 3 = (24ty + 4y) + (-18t - 3) \quad \text{Group terms with common factors.}
\]

\[
4y(6t + 1) + (-3)(6t + 1) \quad \text{Factor GCF from each group.}
\]

\[
(6t + 1)(4y - 3) \quad \text{Common factor } (6t+1), \text{ Dist. Prop.}
\]

27. \( 3dt - 21d + 35 - 5t \)

**SOLUTION:**

\[
3dt - 21d + 35 - 5t = (3dt - 5t) + (-21d + 35) \quad \text{Group terms with common factors.}
\]

\[
t(3d - 5) + (-7)(3d - 5) \quad \text{Factor GCF from each group.}
\]

\[
(3d - 5)(t - 7) \quad \text{Common factor } (3d-5), \text{ Dist. Prop.}
\]

28. \( 8r^2 + 12r \)

**SOLUTION:**

\[
8r^2 + 12r = 4r(2r) + 4r(3)
\]

\[
= 4r(2r + 3)
\]
8-5 Using the Distributive Property

29. \(21b - 15a\)

**SOLUTION:**
Sometimes you need to reorder the terms so that there are common factors in each group. It is important that after 2nd step, there is a common factor. If not, go back and re-group the original terms and factor GCF’s again.

\[
21b - 15a = (21b - 15a) - 0
\]

\[
= (21b - 35b) + (-35a + 15a)
\]

\[
= 7b(3t - 5) + (-1)(3t - 5)
\]

\[
= (3t - 5)(7b - 1)
\]

30. \(vp + 12v + 8p + 96\)

**SOLUTION:**
Sometimes you need to reorder the terms so that there are common factors in each group. It is important that after 2nd step, there is a common factor. If not, go back and re-group the original terms and factor GCF’s again.

\[
vp + 12v + 8p + 96
\]

\[
= (vp + 8p) + (12v + 96)
\]

\[
= p(v + 8) + 12(v + 8)
\]

\[
= (v + 8)(p + 12)
\]

31. \(5br - 25b + 2r - 10\)

**SOLUTION:**

\[
5br - 25b + 2r - 10
\]

\[
= (5br - 25b) + (2r - 10)
\]

\[
= 5b(r - 5) + 2(r - 5)
\]

\[
= (r - 5)(5b + 2)
\]

32. \(2nu - 8u + 3n - 12\)

**SOLUTION:**

\[
2nu - 8u + 3n - 12
\]

\[
= (2nu + 3n) + (-8u - 12)
\]

\[
= n(2u + 3) + (-4)(2u + 3)
\]

\[
= (2u + 3)(n - 4)
\]

33. \(5g^2 + g^2f + 15gf\)

**SOLUTION:**

\[
5g^2 + g^2f + 15gf = g(f(5f) + f(g) + f(15))
\]

\[
= g(f(5f + g + 15))
\]
8-5 Using the Distributive Property

34. \(rp - 9r + 9p - 81\)

\[\text{SOLUTION:}\]
Sometimes you need to reorder the terms so that there are common factors in each group. It is important that after 2nd step, there is a common factor. If not, go back and re-group the original terms and factor GCF’s again.

\[rp - 9r + 9p - 81\]
\[= (rp + 9p) + (-9r - 81)\] Group terms with com factors.
\[= p(r + 9) + (-9)(r + 9)\] Factor GCF from each group.
\[= (r + 9)(p - 9)\] Common factor (r+9), Dist. Prop

35. \(27cd^2 - 18c^2d^2 + 3cd\)

\[\text{SOLUTION:}\]
\[27cd^2 - 18c^2d^2 + 3cd\]
\[= 3cd(9d^2) + 3cd(-6cd) + 3cd(1)\]
\[= 3cd(9d^2 - 6cd + 1)\]

36. \(18r^3 + 12r^2 - 6r^2t\)

\[\text{SOLUTION:}\]
\[18r^3t + 12r^2t - 6r^2t = 6r^2t(3rt) + 6r^2t(2t) + 6r^2t(-1)\]
\[= 6r^2t(3rt + 2t - 1)\]

37. \(48tu - 90t + 32u - 60\)

\[\text{SOLUTION:}\]
\[48tu - 90t + 32u - 60\]
\[= (48tu - 90t) + (32u - 60)\] Group terms w/ com factors.
\[= 6t(8u - 15) + 4(8u - 15)\] Factor GCF from each group.
\[= (8u - 15)(6t + 4)\] Common factor (8u - 15), Dist. Prop
\[= 2(8u - 15)(3t + 2)\] Factor GCF of 2.

38. \(16gh + 24g - 2h - 3\)

\[\text{SOLUTION:}\]
Sometimes you need to reorder the terms so that there are common factors in each group. It is important that after 2nd step, there is a common factor. If not, go back and re-group the original terms and factor GCF’s again.

\[16gh + 24g - 2h - 3\]
\[= (16gh - 2h) + (24g - 3)\] Group terms w/ com factors.
\[= 2h(8g - 1) + 3(8g - 1)\] Factor GCF from each group.
\[= (8g - 1)(2h + 3)\] Common factor (8g - 1), Dist. Prop
8-5 Using the Distributive Property

Solve each equation. Check your solutions.

39. $3b(9b - 27) = 0$

**SOLUTION:**
Since 0 is on one side of the equation and the other side is in factor form, apply the Zero Product Property and set each factor equal to 0. Solve each of the resulting equations.

$3b(9b - 27) = 0$

$3b = 0$ or $9b - 27 = 0$

$b = 0$ or $9b = 27$

The roots are 0 and 3. Check by substituting 0 and 3 in for $b$ in the original equation.

$3b(9b - 27) = 0$

$3(0) [9(0) - 27] = 0$ or $3(3) [9(3) - 27] = 0$

$0(0 - 27) = 0$ or $9(27 - 27) = 0$

$0(- 27) = 0$ or $9(0) = 0$

$0 = 0\checkmark$ or $0 = 0\checkmark$

So, the solutions are 0 and 3.

40. $2n(3n + 3) = 0$

**SOLUTION:**
Since 0 is on one side of the equation and the other side is in factor form, apply the Zero Product Property and set each factor equal to 0. Solve each of the resulting equations.

$2n(3n + 3) = 0$

$2n = 0$ or $3n + 3 = 0$

$n = 0$ or $3n = -3$

The roots are 0 and $-1$. Check by substituting 0 and $-1$ in for $n$ in the original equation.

$2n(3n + 3) = 0$

$2(0) [3(0) + 3] = 0$ or $2(-1) [3(-1) + 3] = 0$

$0[0 + 3] = 0$ or $-2[-3 + 3] = 0$

$0(3) = 0$ or $-2(0) = 0$

$0 = 0\checkmark$ or $0 = 0\checkmark$

So, the solutions are 0 and $-1$. 

---
8-5 Using the Distributive Property

41. \((8z + 4)(5z + 10) = 0\)

**SOLUTION:**

Since 0 is on one side of the equation and the other side is in factor form, apply the Zero Product Property and set each factor equal to 0. Solve each of the resulting equations.

\((8z + 4)(5z + 10) = 0\)

\[ \begin{align*}
8z + 4 &= 0 \quad \text{or} \quad 5z + 10 &= 0 \\
8z &= -4 \quad &5z &= -10 \\
z &= -\frac{1}{2} \quad &z &= -2 \\
\end{align*} \]

The roots are \(-\frac{1}{2}\) and 2. Check by substituting \(-\frac{1}{2}\) and 2 in for \(z\) in the original equation.

\[
\begin{align*}
(8\left(-\frac{1}{2}\right) + 4)(5\left(-\frac{1}{2}\right) + 10) &= 0 \\
[8(-2) + 4][5(-2) + 10] &= 0 \\
(-16 + 4)(-10 + 10) &= 0 \\
(-12)(0) &= 0 \\
0 &= 0 \checkmark
\end{align*}
\]

So, the solutions are \(-\frac{1}{2}\) and 2.

42. \((7x + 3)(2x - 6) = 0\)

**SOLUTION:**

Since 0 is on one side of the equation and the other side is in factor form, apply the Zero Product Property and set each factor equal to 0. Solve each of the resulting equations.

\((7x + 3)(2x - 6) = 0\)

\[ \begin{align*}
7x + 3 &= 0 \quad \text{or} \quad 2x - 6 &= 0 \\
7x &= -3 \quad &2x &= 6 \\
x &= -\frac{3}{7} \quad &x &= 3 \\
\end{align*} \]

The roots are \(-\frac{3}{7}\) and 3. Check by substituting \(-\frac{3}{7}\) and 3 in for \(x\) in the original equation.

\[
\begin{align*}
(7\left(-\frac{3}{7}\right) + 3)(2\left(-\frac{3}{7}\right) - 6) &= 0 \\
[7\left(-\frac{3}{7}\right) + 3][2\left(-\frac{3}{7}\right) - 6] &= 0 \\
[21 + 3][6 - 6] &= 0 \\
(24)(0) &= 0 \\
0 &= 0 \checkmark
\end{align*}
\]

So, the solutions are \(-\frac{3}{7}\) and 3.
43. $b^2 = -3b$

**SOLUTION:**
Rewrite the equation so one side has 0 and the other side is in factor form. Next, apply the Zero Product Property by setting each factor equal to 0. Solve each of the resulting equations.

\[ b^2 = -3b \]
\[ b^2 + 3b = 0 \]
\[ b(b + 3) = 0 \]
\[ b = 0 \quad \text{or} \quad b + 3 = 0 \]

The roots are 0 and $-3$. Check by substituting 0 and $-3$ in for $b$ in the original equation.

\[ (0)^2 = 0 \quad \text{or} \quad (-3)^2 = 9 \]

So, the solutions are 0 and $-3$.

44. $a^2 = 4a$

**SOLUTION:**
Rewrite the equation so one side has 0 and the other side is in factor form. Next, apply the Zero Product Property by setting each factor equal to 0. Solve each of the resulting equations.

\[ a^2 = 4a \]
\[ a^2 - 4a = 0 \]
\[ a(a - 4) = 0 \]
\[ a = 0 \quad \text{or} \quad a - 4 = 0 \]

The roots are 0 and 4. Check by substituting 0 and 4 in for $a$ in the original equation.

\[ (0)^2 = 0 \quad \text{or} \quad (4)^2 = 16 \]

So, the solutions are 0 and 4.
8-5 Using the Distributive Property

45. **CCSS SENSE-MAKING** Use the drawing shown.

![Diagram](image)

a. Write an expression in factored form to represent the area of the blue section.
b. Write an expression in factored form to represent the area of the region formed by the outer edge.
c. Write an expression in factored form to represent the yellow region.

**SOLUTION:**

a. $ab$
b. $(a + 6)(b + 6)$
c. $(a + 6)(b + 6) - ab = ab + 6a + 6b + 36 - ab$
   
   $= 6a + 6b + 36$
   
   $= 6(a + b + 6)$

46. **FIREWORKS** A ten-inch fireworks shell is fired from ground level. The height of the shell in feet is given by the formula $h = 263t - 16t^2$, where $t$ is the time in seconds after launch.

a. Write the expression that represents the height in factored form.
b. At what time will the height be 0? Is this answer practical? Explain.
c. What is the height of the shell 8 seconds and 10 seconds after being fired?
d. At 10 seconds, is the shell rising or falling?

**SOLUTION:**

a. $263t - 16t^2 = t(263 - 16t)$
b.

$t = 0$ or $263 - 16t = 0$

$t = 0$ or $263 = 16t$

$16.4375 = t$

The answer is practical because the shell starts at ground level where $t = 0$ and $h = 0$ and is in the air for 16.4375 seconds before landing on the ground ($h = 0$) again.

c. $h = 263(8) - 16(8)^2$

$= 2104 - 1024$

$= 1080$

After 8 seconds the shell is 1080 ft. high.

$h = 263(10) - 16(10)^2$

$= 2630 - 1600$

$= 1030$

After 10 seconds, the shell is 1030 ft. high.

d. Because the height is lower at 10 seconds than it is at 8 seconds, the shell has begun to fall.
47. ARCHITECTURE The frame of a doorway is an arch that can be modeled by the graph of the equation \( y = -3x^2 + 12x \), where \( x \) and \( y \) are measured in feet. On a coordinate plane, the floor is represented by the \( x \)-axis.

a. Make a table of values for the height of the arch if \( x = 0, 1, 2, 3, \) and \( 4 \) feet.

\[
\begin{array}{|c|c|c|}
\hline
x & y = -3x^2 + 12x & y \\
\hline
0 & -3(0)^2 + 12(0) & 0 \\
1 & -3(1)^2 + 12(1) & 9 \\
2 & -3(2)^2 + 12(2) & 12 \\
3 & -3(3)^2 + 12(3) & 9 \\
4 & -3(4)^2 + 12(4) & 0 \\
\hline
\end{array}
\]

b. Plot the points from the table on a coordinate plane and connect the points to form a smooth curve to represent the arch.

c. How high is the doorway?

**SOLUTION:**

b. 

c. The maximum point occurs at \( x = 2 \) with a height of 12. Thus, the doorway is 12 ft high because 12 ft is the maximum height.

48. RIDES Suppose the height of a rider after being dropped can be modeled by \( h = -16t^2 - 96t + 160 \), where \( h \) is the height in feet and \( t \) is time in seconds.

a. Write an expression to represent the height in factored form.

b. From what height is the rider initially dropped?

c. At what height will the rider be after 3 seconds of falling? Is this possible? Explain.

**SOLUTION:**

a. \(-16t^2 - 96t + 160 = 16(-t^2 - 6t + 10)\)

b. The constant in the expression is 160, so the rider is initially dropped from 160 ft.

c. 
\[
\begin{align*}
h &= -16t^2 - 96t + 160 \\
&= -16(3)^2 - 96(3) + 160 \\
&= -144 - 288 + 160 \\
&= -272
\end{align*}
\]

No, this is not possible. The rider cannot be a negative number of feet in the air.
8-5 Using the Distributive Property

49. ARCHERY  The height \( h \) in feet of an arrow can be modeled by the equation \( h = 64t - 16t^2 \), where \( t \) is time in seconds. Ignoring the height of the archer, how long after it is released does it hit the ground?

**SOLUTION:**
\[
64t - 16t^2 = 0
\]
\[
8t(8-2t) = 0
\]
\[
8t = 0 \quad \text{or} \quad 8-2t = 0
\]
\[
t = 0 \quad \text{or} \quad 8 = 2t
\]
\[
t = 0 \quad \text{or} \quad 4 = t
\]
It takes the arrow 4 seconds to hit the ground.

50. TENNIS  A tennis player hits a tennis ball upward with an initial velocity of 80 feet per second. The height \( h \) in feet of the tennis ball can be modeled by the equation \( h = 80t - 16t^2 \), where \( t \) is time in seconds. Ignoring the height of the tennis player, how long does it take the ball to hit the ground?

**SOLUTION:**
\[
80t - 16t^2 = 0
\]
\[
8t(10-2t) = 0
\]
\[
8t = 0 \quad \text{or} \quad 10-2t = 0
\]
\[
t = 0 \quad \text{or} \quad 10 = 2t
\]
\[
t = 0 \quad \text{or} \quad 5 = t
\]
It takes the ball 5 seconds to hit the ground.

51. MULTIPLE REPRESENTATIONS  In this problem, you will explore the box method of factoring. To factor \( x^2 + x - 6 \), write the first term in the top left-hand corner of the box, and then write the last term in the lower right-hand corner.

\[
\begin{array}{c|c|c}
? & x^2 & ? \\
? & ? & -6 \\
\end{array}
\]

a. **ANALYTICAL**  Determine which two factors have a product of \(-6\) and a sum of 1.

b. **SYMBOLIC**  Write each factor in an empty square in the box. Include the positive or negative sign and variable.

c. **ANALYTICAL**  Find the factor for each row and column of the box. What are the factors of \( x^2 + x - 6 \)?

d. **VERBAL**  Describe how you would use the box method to factor \( x^2 - 3x - 40 \).

**SOLUTION:**

a.
Make a table with the different factors of \(-6\). Find the sum and product of each pair of factors.

<table>
<thead>
<tr>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Product of Factors</th>
<th>Sum of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-6</td>
<td>-6</td>
<td>-5</td>
</tr>
<tr>
<td>-1</td>
<td>6</td>
<td>-6</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>-3</td>
<td>-6</td>
<td>-1</td>
</tr>
<tr>
<td>-2</td>
<td>3</td>
<td>-6</td>
<td>1</td>
</tr>
</tbody>
</table>
8-5 Using the Distributive Property

The factors are 3 and –2 since 3 + (–2) = 1 and 3(–2) = –6.

b. 

\[
\begin{array}{ccc}
\phantom{-}x^2 & +3x \\
-2x & -6
\end{array}
\]

\((x + 3)(x - 2)\)

d. Place \(x^2\) in the top left-hand corner and place –40 in the lower right-hand corner. Then determine which two factors have a product of –40 and a sum of –3. Then place these factors in the box. Then find the factor of each row and column. The factors will be listed on the very top and far left of the box.

\[
\begin{array}{ccc}
x & -8 \\
x^2 & -8x \\
+5 & 40
\end{array}
\]
52. **CCSS CRITIQUE** Hernando and Rachel are solving $2m^2 = 4m$. Is either of them correct? Explain your reasoning.

**Hernando**

\[
\begin{align*}
2m^2 &= 4m \\
2m^2 - 4m &= 0 \\
2m(m - 2) &= 0 \\
2m &= 0 \text{ or } m - 2 = 0 \\
m &= 0 \text{ or } 2
\end{align*}
\]

**Rachel**

\[
\begin{align*}
2m^2 &= 4m \\
2m^2 - 4m &= 0 \\
2m(m - 2) &= 0 \\
2m &= 0 \text{ or } m = 2 \\
m &= 0 \text{ or } 2
\end{align*}
\]

**SOLUTION:**

Hernando divided one side by $m$ and the other by $2m$. The Division Property of Equality states that each side must be divided by the same quantity. Also, since $m$ is a variable it could equal 0. Division by 0 does not produce an equivalent equation. Rachel has applied the Zero Product Property by rewriting the equation so that one side has zero and the other side is in factor form. Then she set each factor equal to 0 and solved. Rachel is correct.

53. **CHALLENGE** Given the equation $(ax + b)(ax - b) = 0$, solve for $x$. What do we know about the values of $a$ and $b$?

**SOLUTION:**

\[
\begin{align*}
ax + b &= 0 & ax - b &= 0 \\
ax &= -b & ax &= b \\
\frac{ax}{a} &= \frac{-b}{a} & \frac{ax}{a} &= \frac{b}{a} \\
x &= -\frac{b}{a} & x &= \frac{b}{a}
\end{align*}
\]

If $a = 0$ and $b = 0$, then all real numbers are solutions. If $a \neq 0$, then the solutions are $-\frac{b}{a}$ and $\frac{b}{a}$.

54. **OPEN ENDED** Write a four-term polynomial that can be factored by grouping. Then factor the polynomial.

**SOLUTION:**

Write 4 monomials where there is a GCF for each group. Once the GCF is factored out, there should be another common factor.

\[
\begin{align*}
x^2 + 2xy + 3x + 6y &= (x^2 + 2xy) + (3x + 6y) \\
&= x(x + 2y) + 3(x + 2y) \\
&= (x + 3)(x + 2y)
\end{align*}
\]
8-5 Using the Distributive Property

55. REASONING  Given the equation $c = a^2 - ab$, for what values of $a$ and $b$ does $c = 0$?

**SOLUTION:**
Let $c = 0$, factor and solve.

\[a^2 - ab = 0\]
\[a(a - b) = 0\]
\[a = 0 \quad \text{or} \quad a = b\]
Then $c = 0$ when $a = 0$ or $a = b$ for any real values of $a$ and $b$.

56. WRITING IN MATH  Explain how to solve a quadratic equation by using the Zero Product Property.

**SOLUTION:**
Rewrite the equation to have zero on one side of the equals sign. Then factor the other side. Set each factor equal to zero, and then solve each equation. For example solve $2x^2 - 5x = 3$.

\[2x^2 - 5x = 3\]
\[2x^2 - 5x - 3 = 3 - 3\]
\[2x^2 - 5x - 3 = 0\]
\[(2x + 1)(x - 3) = 0\]
\[x = \frac{3x - 1}{2}\]

57. Which is a factor of $6x^2 - 3x - 2 + 4x$?
A $2x + 1$
B $3x - 2$
C $z + 2$
D $2z - 1$

**SOLUTION:**
\[6x^2 - 3x - 2 + 4x = (6x^2 - 3x) + (4x - 2)\]
\[= 3z(2z - 1) + 2(2z - 1)\]
\[= (2z - 1)(3z + 2)\]
The correct choice is D.
8-5 Using the Distributive Property

58. **PROBABILITY**  Hailey has 10 blocks: 2 red, 4 blue, 3 yellow, and 1 green. If Hailey chooses one block, what is the probability that it will be either red or yellow?

   **F** \( \frac{3}{10} \)
   **G** \( \frac{1}{5} \)
   **H** \( \frac{1}{2} \)
   **J** \( \frac{7}{10} \)

   **SOLUTION:**
   The total number of blocks is 10 and the number of red or yellow blocks is \( 2 + 3 \), or 5. So the probability of choosing either a red or a yellow block is \( \frac{5}{10} \) or \( \frac{1}{2} \).
   The correct choice is H.

59. **GRIDDED RESPONSE**  Cho is making a 140-inch by 160-inch quilt with quilt squares that measure 8 inches on each side. How many will be needed to make the quilt?

   **SOLUTION:**
   The area of the quilt is \( 140 \times 160 \), or 22,400. The area of each square is \( 8 \times 8 \), or 64. To find the number of squares needed, divide the area of the quilt by the area of each square: \( 22,400 \div 64 = 350 \). So, 350 squares will be needed to make the quilt.

60. **GEOMETRY**  The area of the right triangle shown is \( 5h \) square centimeters. What is the height of the triangle?

   **A** 2 cm
   **B** 5 cm
   **C** 8 cm
   **D** 10 cm

   **SOLUTION:**
   \[
   A = \frac{1}{2}bh \\
   5h = \frac{1}{2}h(2h) \\
   5h = h^2 \\
   0 = h^2 - 5h \\
   0 = h(h - 5) \\
   h = 0 \quad \text{or} \quad h - 5 = 0 \\
   h = 5
   \]
   The height of the triangle is \( 2h \), or \( 2(5) = 10 \) cm.
   The correct choice is D.
8-5 Using the Distributive Property

61. GENETICS  Brown genes $B$ are dominant over blue genes $b$. A person with genes $BB$ or $Bb$ has brown eyes. Someone with genes $bb$ has blue eyes. Elisa has brown eyes with $Bb$ genes, and Bob has blue eyes. Write an expression for the possible eye coloring of Elisa and Bob’s children. Determine the probability that their child would have blue eyes.

**SOLUTION:**
Bob has blue eyes so he must have $bb$ genes. Elisa has brown eyes so she could have $BB$, $Bb$, or $bB$ genes. Their children could get only a $b$ gene from Bob but either a $B$ or $b$ gene from Elisa. The following table shows the combinations if each child would receive half of each parent’s genes.

<table>
<thead>
<tr>
<th></th>
<th>$B$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>$Bb$</td>
<td>$bb$</td>
</tr>
<tr>
<td>$B$</td>
<td>$Bb$</td>
<td>$bb$</td>
</tr>
</tbody>
</table>

So, an expression for the possible eye coloring of Elisa and Bob’s children is: $0.5Bb + 0.5b^2$.

If $B = 0$ and $b = 1$, then

$0.5Bb + 0.5b^2 = 0.5(0)(1) + 0.5(1)^2$

$= 0 + 0.5$

$= 0.5$

$= \frac{1}{2}$

Therefore, the probability that a child of Elisa and Bob would have blue eyes is $\frac{1}{2}$.

Find each product.

62. $n(n^2 - 4n + 3)$

**SOLUTION:**

$n\left(n^2 - 4n + 3\right)$  
Original expression

$= n\left(n^2\right) - n(4n) + n(3)$  
Distributive Property

$= n^3 - 4n^2 + 3n$  
Multiply.

63. $2b(b^2 + b - 5)$

**SOLUTION:**

$2b\left(b^2 + b - 5\right)$  
Orig express

$= (2b)\left(b^2\right) + (2b)(b)$

$= -2b(5)$  
Dist. Prop.

$= 2b^3 + 2b^2 - 10b$  
Multiply.
8-5 Using the Distributive Property

64. \(-c(4c^2 + 2c - 2)\)

\[
\text{SOLUTION:} \\
-\left\{\begin{array}{l}
\end{array}\right. \\
= c \left(\begin{array}{c}
4c^2 + 2c - 2
\end{array}\right) \\
\end{align*}
\]

65. \(-4x(x^2 + x^2 + 2x - 1)\)

\[
\text{SOLUTION:} \\
-4x \left(\begin{array}{c}
4c^2 + 2c - 2
\end{array}\right) \\
\end{align*}
\]

66. \(2ab(4a^2b + 2ab - 2b^2)\)

\[
\text{SOLUTION:} \\
2ab \left(\begin{array}{c}
4a^2b + 2ab - 2b^2
\end{array}\right) \\
\end{align*}
\]

67. \(-3xy(x^2 + xy + 2y^2)\)

\[
\text{SOLUTION:} \\
-3xy \left(\begin{array}{c}
x^2 + xy + 2y^2
\end{array}\right) \\
\end{align*}
\]
8-5 Using the Distributive Property

Simplify.
68. \((ab^4)(ab^2)\)

**SOLUTION:**
\[(ab^4)(ab^2) = a^{(1+1)}b^{(4+2)} = a^2b^6\]

69. \((p^5r^4)(p^2r)\)

**SOLUTION:**
\[(p^5r^4)(p^2r) = p^{(5+2)}r^{(4+1)} = p^7r^5\]

70. \((-7c^3d^4)(4cd^3)\)

**SOLUTION:**
\[(-7c^3d^4)(4cd^3) = (-7 \cdot 4)c^{(3+1)}d^{(4+3)} = -28c^4d^7\]

71. \((9xy^7)^2\)

**SOLUTION:**
\[(9xy^7)^2 = 9^2(x)^2(y^7)^2 = 81x^2y^{(7\cdot2)} = 81x^2y^{14}\]

72. \[\left[(3^4)^7\right]^2\]

**SOLUTION:**
\[\left[(3^4)^7\right]^2 = \left[(9)^4\right]^2 = (6,561)^2 = 43,046,721\]

73. \[\left[(4^3)^5\right]^3\]

**SOLUTION:**
\[\left[(4^3)^5\right]^3 = \left[(16)^3\right]^2 = (4,096)^2 = 16,777,216\]
8-5 Using the Distributive Property

74. BASKETBALL  In basketball, a free throw is 1 point, and a field goal is either 2 or 3 points. In a season, Tim Duncan of the San Antonio Spurs scored a total of 1342 points. The total number of 2-point field goals and 3-point field goals was 517, and he made 305 of the 455 free throws that he attempted. Find the number of 2-point field goals and 3-point field goals Duncan made that season.

**SOLUTION:**
Let \( t \) = the number of 3-point field goals Duncan made. His total number of points can be found by the sum of the points earned by his 3-point field goals, his 2-point field goals, and his free throws:

\[
1342 = 3t + 2(517 - t) + 305
\]

\[
1037 = 3t + 1034 - 2t
\]

\[
3 = t
\]

\[
517 - 3 = 514
\]

Duncan made three 3-point field goals and 514 2-point field goals that season.

Solve each inequality. Check your solution.

75. \( 3y - 4 > -37 \)

**SOLUTION:**

\[
3y - 4 > -37
\]

\[
3y - 4 + 4 > -37 + 4
\]

\[
3y > -33
\]

\[
\frac{3y}{3} > \frac{-33}{3}
\]

\[
y > -11
\]

The solution is \{y\} \( y > -11 \).

Check the solution by substituting a number greater than \(-11\) for \( y \) in the equation.

\[
3(1) - 4 > -37
\]

\[
-1 > -37
\]

The solution checks.

76. \( -5q + 9 > 24 \)

**SOLUTION:**

\[
-5q + 9 > 24
\]

\[
-5q + 9 - 9 > 24 - 9
\]

\[
-5q < 15
\]

\[
\frac{-5q}{-5} < \frac{15}{-5}
\]

\[
q < -3
\]

The solution is \{q\} \( q < -3 \).

Check the solution by substituting a number less than \(-3\) for \( q \) in the equation.

\[
-5(-4) + 9 > 24
\]

\[
20 + 9 > 24
\]

\[
29 > 24
\]

The solution checks.
77. $-2k + 12 < 30$

**SOLUTION:**

$$-2k + 12 < 30$$

$$-2k + 12 - 12 < 30 - 12$$

$$-2k > 18$$

$$\frac{-2k}{-2} > \frac{18}{-2}$$

$$k > -9$$

The solution is \( \{ k | k > -9 \} \).

Check the solution by substituting a number greater than $-9$ for $k$ in the equation.

\( -2(1) + 12 < 30 \)

\( -2 + 12 < 30 \)

\( 10 < 30 \)

The solution checks.

78. $5q + 7 \leq 3(q + 1)$

**SOLUTION:**

$$5q + 7 \leq 3(q + 1)$$

$$5q + 7 \leq 3q + 3$$

$$5q - 3q + 7 \leq 3q - 3q + 3$$

$$2q + 7 \leq 3$$

$$2q + 7 - 7 \leq 3 - 7$$

$$2q \leq -4$$

$$\frac{2q}{2} \leq \frac{-4}{2}$$

$$q \leq -2$$

The solution is \( \{ q | q \leq -2 \} \).

Check the solution by substituting a number less than or equal to $-2$ for $q$ in the equation.

$$5(-2) + 7 \leq 3(-2 + 1)$$

$$-10 + 7 \leq 3(-1)$$

$$-3 \leq -3$$

The solution checks.
8-5 Using the Distributive Property

79. \( \frac{z}{4} + 7 \geq -5 \)

**SOLUTION:**

\[
\frac{z}{4} + 7 \geq -5 \\
\frac{z}{4} + 7 - 7 \geq -5 - 7 \\
\frac{z}{4} \geq -12 \\
4(\frac{z}{4}) \geq 4(-12) \\
z \geq -48
\]

The solution is \( \{z \geq -48\} \).
Check the solution by substituting a number greater than or equal to -48 for \( z \) in the equation.

\[
\frac{4 \cdot 8}{4} + 7 \geq -5 \\
1 + 7 \geq -5 \\
8 \geq -5
\]

The solution checks.

80. \( 8c - (c - 5) > c + 17 \)

**SOLUTION:**

\[
8c - (c - 5) > c + 17 \\
8c - c + 5 > c + 17 \\
7c - c + 5 > c - c + 17 \\
6c + 5 > 17 \\
6c + 5 - 5 > 17 - 5 \\
6c > 12 \\
\frac{6c}{6} > \frac{12}{6} \\
c > 2
\]

The solution is \( \{c | c > 2\} \).
Check the solution by substituting a number greater than 2 for \( c \) in the equation.

\[
8(5) - (5 - 5) \geq 5 + 17 \\
40 \geq 22
\]

The solution checks.

**Find each product.**

81. \((a + 2)(a + 5)\)

**SOLUTION:**

\[
(a + 2)(a + 5) = a(a) + a(5) + 2(a) + 2(5) \\
= a^2 + 5a + 2a + 10 \\
= a^2 + 7a + 10
\]
8-5 Using the Distributive Property

82. \((d + 4)(d + 10)\)

**SOLUTION:**
\[
(d + 4)(d + 10) = d(d) + d(10) + 4(d) + 4(10)
\]
\[
= d^2 + 10d + 4d + 40
\]
\[
= d^2 + 14d + 40
\]

83. \((z - 1)(z - 8)\)

**SOLUTION:**
\[
(z - 1)(z - 8) = z(z) + z(-8) + (-1)(z) + (-1)(-8)
\]
\[
= z^2 + (-8z) + (-1z) + 8
\]
\[
= z^2 - 9z + 8
\]

84. \((c + 9)(c - 3)\)

**SOLUTION:**
\[
(c + 9)(c - 3) = c(c) + c(-3) + 9(c) + 9(-3)
\]
\[
= c^2 + (-3c) + 9c + (-27)
\]
\[
= c^2 + 6c - 27
\]

85. \((x - 7)(x - 6)\)

**SOLUTION:**
\[
(x - 7)(x - 6) = x(x) + x(-6) + (-7)x + (-7)(-6)
\]
\[
= x^2 + (-6x) + (-7x) + 42
\]
\[
= x^2 - 13x + 42
\]

86. \((g - 2)(g + 11)\)

**SOLUTION:**
\[
(g - 2)(g + 11) = g(g) + g(11) + (-2)(g) + (-2)(11)
\]
\[
= g^2 + 11g + (-2g) + (-22)
\]
\[
= g^2 + 9g - 22
\]