Choose the word or term that best completes each sentence.

1. $7xy^4$ is an example of a(n)___________ .
   
   **SOLUTION:**
   A product of a number and variables is a monomial.

2. The __________ of 95,234 is $10^5$.
   
   **SOLUTION:**
   95,234 is almost 100,000 or $10^5$, so $10^5$ is its order of magnitude.

3. 2 is a(n) __________ of 8.
   
   **SOLUTION:**
   Since $8 = 2^3$, 2 is the cube root of 8.

4. The rules for operations with exponents can be extended to apply to expressions with a(n) __________ such as $\frac{2}{3}$.
   
   **SOLUTION:**
   The exponent $\frac{2}{3}$ is a rational exponent.

5. A number written in is of the form $a \times 10^n$, where $1 \leq a < 10$ and $n$ is an integer.
   
   **SOLUTION:**
   A number written as $a \times 10^n$, where $1 \leq a < 10$ and $n$ is an integer, is written in scientific notation.

6. $f(x) = 3^x$ is an example of a(n) _____________.
   
   **SOLUTION:**
   A function in the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$ is an exponential function.

7. $a_1 = 4$ and $a_n = 3a_{n-1} + 16$, if $n \geq 2$, is a(n) ____________ for the sequence 4, −8, −20, −32, ... .
   
   **SOLUTION:**
   A formula that gives the first term of a sequence and tells you how to find the next term when you know the preceding term is called a recursive formula.

8. $2^{3x - 1} = 16$ is an example of a(n) _____________.
   
   **SOLUTION:**
   An equation in which the variable occurs as exponent is an exponential equation.

9. The equation for ___________ is $y = C(1 - r)^t$.
   
   **SOLUTION:**
   Since $0 < 1 - r < 1$, this is an example of exponential decay.
10. If $a^n = b$ for a positive integer $n$, then $a$ is a(n) ________________ of $b$.

**SOLUTION:**
If $a^n = b$ for a positive integer $n$, then $a$ is an $n$th root of $b$.

**Simplify each expression.**

11. $x \cdot x^3 \cdot x^5$

**SOLUTION:**
$x \cdot x^3 \cdot x^5 = (x \cdot x^3) \cdot x^5$  
Associative Property

$= (x^{1+3}) \cdot x^5$  
Product of Powers

$= x^4 \cdot x^5$  
Simplify.

$= x^{4+5}$  
Product of Powers

$= x^9$  
Simplify.

12. $(2xy)(-3x^2y^5)$

**SOLUTION:**
$(2xy)(-3x^2y^5) = [2 \cdot (-3)] (x \cdot x^2)(y \cdot y^5)$  
Group the coefficients and the variables.

$= -6(x^{1+2})(y^{1+5})$  
Product of Powers

$= -6x^3y^6$  
Simplify.

13. $(-4ab^4)(-5a^5b^2)$

**SOLUTION:**
$(-4ab^4)(-5a^5b^2) = [-4 \cdot (-5)] (a \cdot a^5)(b^4 \cdot b^2)$  
Group the coefficients and the variables

$= 20(a^{1+5})(b^{4+2})$  
Product of Powers

$= 20a^6b^6$  
Simplify.

14. $(6x^3y^2)^2$

**SOLUTION:**
$(6x^3y^2)^2 = 6^2(x^3)^2(y^2)^2$  
Power of a Product

$= 36(x^{3 \cdot 2})(y^{2 \cdot 2})$  
Power of a Power

$= 36x^6y^4$  
Simplify.
15. \[{(2r^3t^3)}^2\]

**SOLUTION:**

\[
[\left(2r^3t^3\right)^2] = [2^2\left(r^3\right)^2\left(t^3\right)^2] \quad \text{Power of a Product}
\]

\[
= [8\left(r^3\right)^2\left(t^3\right)^2] \quad \text{Power of a Power}
\]

\[
= (8r^9t^6)^2 \quad \text{Simplify}
\]

\[
= 8^2(r^9)^2(t^3)^2 \quad \text{Power of a Product}
\]

\[
= 64\left(r^{18}\right)^2\left(t^{6}\right)^2 \quad \text{Power of a Power}
\]

\[
= 64r^{18}t^6 \quad \text{Simplify}
\]

16. \((-2u^3)(5u)\)

**SOLUTION:**

\[
(-2u^3)(5u) = (-2 \cdot 5)(u^3 \cdot u) \quad \text{Group the coefficients and the variables}
\]

\[
= -10u^{3+1} \quad \text{Product of Powers}
\]

\[
= -10u^4 \quad \text{Simplify}
\]

17. \((2x^2)^3 \left(x^3\right)^3\)

**SOLUTION:**

\[
(2x^2)^3 \left(x^3\right)^3 = \left[2^3\left(x^2\right)^3\right]\left[x^3\right]^3 \quad \text{Power of Product}
\]

\[
= (2^3 \cdot x^6)\left(x^9\right) \quad \text{Power of a Power}
\]

\[
= 8x^{6+9} \quad \text{Simplify}
\]

\[
= 8x^{15} \quad \text{Simplify}
\]

18. \[
\frac{1}{2} \left(2x^3\right)^3
\]

**SOLUTION:**

\[
\frac{1}{2} \left(2x^3\right)^3 = \frac{1}{2} \left[2^3 \cdot x^{3\cdot3}\right] \quad \text{Power of a Power}
\]

\[
= \frac{1}{2} \left(8x^9\right) \quad \text{Simplify}
\]

\[
= 4x^9 \quad \text{Simplify}
\]
19. **GEOMETRY** Use the formula \( V = \pi r^2 h \) to find the volume of the cylinder.

\[
\begin{align*}
V &= \pi r^2 h \\
&= \pi (3x)^2 \left( \frac{5x^2}{2} \right) \\
&= \pi \left( \frac{9x^2}{2} \right) \left( \frac{5x^2}{2} \right) \\
&= \pi \left( \frac{45x^4}{4} \right) \\
&= \frac{45\pi x^4}{4}
\end{align*}
\]

Simplify each expression. Assume that no denominator equals zero.

20. \[ \frac{(3x)^0}{2a} \]

**SOLUTION:**

\[
\begin{align*}
\frac{(3x)^0}{2a} &= \frac{3^0 \cdot x^0}{2a} \\
&= \frac{1 \cdot 1}{2a} \\
&= \frac{1}{2a}
\end{align*}
\]

21. \[ \left( \frac{3xy^3}{2z} \right)^3 \]

**SOLUTION:**

\[
\begin{align*}
\left( \frac{3xy^3}{2z} \right)^3 &= \frac{(3xy)^3}{(2z)^3} \\
&= \frac{3^3 (x)^3 (y^3)^3}{2^3 (z)^3} \\
&= \frac{27x^3 y^9}{8z^3}
\end{align*}
\]
22. \[
\frac{12y^4}{3y^5} = \frac{12}{3} \left( \frac{y^4}{y^5} \right) = 4 \frac{y^4}{y^4} = 4y^0 = 4
\]

**SOLUTION:**

\[
\frac{12y^{-4}}{3y^{-5}} = \frac{12}{3} \left( \frac{y^{-4}}{y^{-5}} \right) = 4y
\]

23. \(a^{-3} b^0 c^6\)

**SOLUTION:**

\[
a^{-3} b^0 c^6 = \frac{1 \cdot c^6}{a^3} = \frac{c^6}{a^3}
\]

24. \[
\frac{-15x^7 y^8 z^4}{-45x^3 y^5 z^3}
\]

**SOLUTION:**

\[
\frac{-15x^7 y^8 z^4}{-45x^3 y^5 z^3} = \left( \frac{-15}{-45} \right) \left( \frac{x^7}{x^3} \right) \left( \frac{y^8}{y^5} \right) \left( \frac{z^4}{z^3} \right) = \frac{1}{3} \left( x^{7-3} y^{8-5} z^{4-3} \right) = \frac{x^4 y^3 z}{3}
\]

Group powers with the same base

Quotient of Powers

Simplify.
25. \( \frac{(3x^{-1})^{-2}}{(3x^2)^{-2}} \)

**SOLUTION:**

\[
\frac{(3x^{-1})^{-2}}{(3x^2)^{-2}} = \frac{3^{-2}(x^{-1})^{-2}}{3^{-2}(x^2)^{-2}} = \frac{3^{-2}x^{-1(-2)}}{3^{-2}x^{2(-2)}} = \frac{(3^{-2})(x^2)}{(3^{-2})(x^{-4})} = \frac{x^2}{x^{-4}} = x^{2+4} = x^6
\]

Simplify.

26. \( \left( \frac{6xy^{11}z^9}{48x^6yz^{-7}} \right)^0 \)

**SOLUTION:**

\[
\left( \frac{6xy^{11}z^9}{48x^6yz^{-7}} \right)^0 = \left( \frac{6^0(6^{0})(x^0)(x^0)(x^{11})(y^0)(y^0)(y^0)(z^9)(z^9)(z^9)}{48^0(6^0)(x^6)(x^6)(x^0)(x^{11})(y^0)(y^0)(y^0)(z^9)(z^9)(z^9)} \right)^0 = \left( \frac{1(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)}{1(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)(1)} \right)^0 = \frac{1}{1} = 1
\]

Simplify.
27. \( \left( \frac{12}{2} \right) \left( \frac{x^2}{y^3} \right) \left( \frac{y^4}{x^1} \right) \)

**SOLUTION:**

\[
\left( \frac{12}{2} \right) \left( \frac{x^2}{y^3} \right) \left( \frac{y^4}{x^1} \right) = 6 \left( \frac{x^{2-1} y^{4-3}}{x^1 y^3} \right) \\
= 6 \left( \frac{x^{1} y^{1}}{x^1 y^3} \right) \\
= 6 \left( x^{1-1} y^{1-3} \right) \\
= 6 x^{-3} y^{-2} \\
= \frac{6}{x^3 y^2} \\
= \frac{1}{a^n}
\]

28. **GEOMETRY** The area of a rectangle is \( 25x^2 y^4 \) square feet. The width of the rectangle is \( 5xy \) feet. What is the length of the rectangle?

**SOLUTION:**

\[
A = l \cdot w \\
25x^2 y^4 = l (5xy) \\
\frac{25x^2 y^4}{5xy} = \frac{l(5xy)}{5xy} \\
(\frac{25}{5}) (\frac{x^2}{x}) (\frac{y^4}{y}) = l \\
5x y^3 = l \\
\]

The length of the rectangle is \( 5xy^3 \) ft.

29. \( \sqrt[3]{343} \)

**SOLUTION:**

\[
\sqrt[3]{343} = \sqrt[3]{7 \cdot 7 \cdot 7} \\
= 7
\]
30. \( \sqrt[6]{729} \)

**SOLUTION:**
\[
\sqrt[6]{729} = \sqrt[6]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = 3
\]

31. \( 625^{\frac{1}{4}} \)

**SOLUTION:**
\[
625^{\frac{1}{4}} = \sqrt[4]{625} = \sqrt[4]{5 \cdot 5 \cdot 5 \cdot 5} = 5
\]

32. \( \left( \frac{8}{27} \right)^{\frac{1}{3}} \)

**SOLUTION:**
\[
\left( \frac{8}{27} \right)^{\frac{1}{3}} = \sqrt[3]{\frac{8}{27}} = \sqrt[3]{\frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3}} = \sqrt[3]{\left( \frac{2}{3} \right)^3} = \frac{2}{3}
\]

33. \( 256^{\frac{3}{4}} \)

**SOLUTION:**
\[
256^{\frac{3}{4}} = \left( \sqrt[4]{256} \right)^3 = \left( \sqrt[4]{4 \cdot 4 \cdot 4 \cdot 4} \right)^3 = 4^3 = 64
\]

34. \( 32^{\frac{2}{5}} \)

**SOLUTION:**
\[
32^{\frac{2}{5}} = \left( \sqrt[5]{32} \right)^2 = \left( \sqrt[5]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} \right)^2 = 2^2 = 4
\]
Study Guide and Review - Chapter 7

35. \( \frac{4}{343^3} \)

**SOLUTION:**
\[
343^\frac{4}{3} = \left(\frac{3}{343}\right)^4 \quad \frac{m}{b^n} = \left(\frac{m}{b}\right)^n
\]
\[
= \left(\frac{3}{7 \cdot 7 \cdot 7}\right)^4 \quad 343 = 7^3
\]
\[
= 7^4 \text{ or } 2401 \quad \text{Simplify.}
\]

36. \( \left(\frac{4}{49}\right)^\frac{3}{2} \)

**SOLUTION:**
\[
\left(\frac{4}{49}\right)^\frac{3}{2} = \left(\frac{4}{49}\right)^3 \quad \frac{m}{b^n} = \left(\frac{m}{b}\right)^n
\]
\[
= \left(\frac{2 \cdot 2 \cdot 2}{7 \cdot 7}\right)^3 \quad \frac{4}{49} = \left(\frac{2}{7}\right)^2
\]
\[
= \left(\frac{2}{7}\right)^3 \text{ or } \frac{8}{343} \quad \text{Simplify.}
\]

**Solve each equation.**

37. \( 6^x = 7776 \)

**SOLUTION:**
\( 6^x = 7776 \) \quad \text{Original equation}
\( 6^x = 6^5 \) \quad \text{Rewrite 7776 as } 6^5.
\( x = 5 \) \quad \text{Power Property of Equality}

Therefore, the solution is 5.

38. \( 4^{4x-1} = 32 \)

**SOLUTION:**
\( 4^{4x-1} = 32 \) \quad \text{Original equation}
\[
\left(2^2\right)^{4x-1} = 2^5 \quad \text{Rewrite 4 as } 2^2 \text{ and 32 as } 2^5.
\]
\[2^{8x-2} = 2^5 \quad \text{Power of a Power}
\]
\[8x - 2 = 5 \quad \text{Power Property of Equality}
\]
\[8x = 7 \quad \text{Add 2 to each side.}
\]
\[x = \frac{7}{8} \quad \text{Divide each side by 8.}
\]

Therefore, the solution is \( \frac{7}{8} \).
Study Guide and Review - Chapter 7

Express each number in scientific notation.
39. 2,300,000

**SOLUTION:**

2,300,000 \rightarrow 2.3 \times 10^6
The decimal point moved 6 places to the left, so \( n = 6 \).

2,300,000 = 2.3 \times 10^6

40. 0.0000543

**SOLUTION:**

0.0000543 \rightarrow 5.43 \times 10^{-5}
The decimal point moved 5 places to the right, so \( n = -5 \).

0.0000543 = 5.43 \times 10^{-5}

41. **ASTRONOMY** Earth has a diameter of about 8000 miles. Jupiter has a diameter of about 88,000 miles. Write in scientific notation the ratio of Earth’s diameter to Jupiter’s diameter.

**SOLUTION:**

Earth: \( 8000 = 8.0 \times 10^3 \)
Jupiter: \( 88,000 = 8.8 \times 10^4 \)

\[
\frac{\text{Earth’s diameter}}{\text{Jupiter’s diameter}} = \frac{8.0 \times 10^3}{8.8 \times 10^4} = \left( \frac{8.0}{8.8} \right) \left( \frac{10^3}{10^4} \right) = 0.91 \times 10^{-1} \\
\approx 9.1 \times 10^{-2}
\]

In scientific notation, the ratio of Earth’s diameter to Jupiter’s diameter is about \( 9.1 \times 10^{-2} \).
Graph each function. Find the \( y \)-intercept, and state the domain and range.

42. \( y = 2^x \)

**SOLUTION:**

Complete a table of values for \(-2 < x < 2\). Connect the points on the graph with a smooth curve.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 2^x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2)</td>
<td>( 2^{-2} )</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>(-1)</td>
<td>( 2^{-1} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>0</td>
<td>( 2^0 )</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>( 2^1 )</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>( 2^2 )</td>
<td>4</td>
</tr>
</tbody>
</table>

The graph crosses the \( y \)-axis at 1. The domain is all real numbers, and the range is all real numbers greater than 0.
43. \( y = 3^x + 1 \)

**SOLUTION:**
Complete a table of values for \(-2 < x < 2\). Connect the points on the graph with a smooth curve.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 3^x + 1 )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>( 3^{-2} + 1 )</td>
<td>( \frac{1}{9} )</td>
</tr>
<tr>
<td>-1</td>
<td>( 3^{-1} + 1 )</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>0</td>
<td>( 3^0 + 1 )</td>
<td>( 2 )</td>
</tr>
<tr>
<td>1</td>
<td>( 3^1 + 1 )</td>
<td>( 4 )</td>
</tr>
<tr>
<td>2</td>
<td>( 3^2 + 1 )</td>
<td>( 10 )</td>
</tr>
</tbody>
</table>

The graph crosses the \( y \)-axis at 2. The domain is all real numbers, and the range is all real numbers greater than 1.
44. \( y = 4^x + 2 \)

**SOLUTION:**
Complete a table of values for \(-2 < x < 2\). Connect the points on the graph with a smooth curve.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 4^x + 2 )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>( 4^{-2} + 2 )</td>
<td>( 2 \frac{1}{16} )</td>
</tr>
<tr>
<td>-1</td>
<td>( 4^{-1} + 2 )</td>
<td>( 2 \frac{1}{4} )</td>
</tr>
<tr>
<td>0</td>
<td>( 4^0 + 2 )</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>( 4^1 + 2 )</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>( 4^2 + 2 )</td>
<td>18</td>
</tr>
</tbody>
</table>

The graph crosses the \( y \)-axis at 3. The domain is all real numbers, and the range is all real numbers greater than 2.
45. \( y = 2^x - 3 \)

**SOLUTION:**
Complete a table of values for \(-2 < x < 2\). Connect the points on the graph with a smooth curve.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 2^x - 3 )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>( 2^{-2} - 3 )</td>
<td>(-2\frac{3}{4})</td>
</tr>
<tr>
<td>-1</td>
<td>( 2^{-1} - 3 )</td>
<td>(-2\frac{1}{2})</td>
</tr>
<tr>
<td>0</td>
<td>( 2^0 - 3 )</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>( 2^1 - 3 )</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>( 2^2 - 3 )</td>
<td>1</td>
</tr>
</tbody>
</table>

The graph crosses the \( y \)-axis at \(-2\). The domain is all real numbers, and the range is all real numbers greater than \(-3\).

46. **BIOLOGY** The population of bacteria in a petri dish increases according to the model \( p = 550(2.7)^{0.008t} \), where \( t \) is the number of hours and \( t = 0 \) corresponds to 1:00 P.M. Use this model to estimate the number of bacteria in the dish at 5:00 P.M.

**SOLUTION:**
If \( t = 0 \) corresponds to 1:00 P.M, then \( t = 4 \) represents 5:00 P.M.

\[
p = 550(2.7)^{0.008\cdot 4} \\
= 550(2.7)^{0.032} \\
= 550(2.7)^{0.032} \\
\approx 568
\]

There will be about 568 bacteria in the dish at 5:00 P.M.
47. Find the final value of $2500 invested at an interest rate of 2% compounded monthly for 10 years.

**SOLUTION:**
Use the equation for compound interest, with $P = 2500$, $r = 0.02$, $n = 12$, and $t = 10$.

\[
A = P \left(1 + \frac{r}{n}\right)^{nt}
\]

\[
= 2500 \left(1 + \frac{0.02}{12}\right)^{120}
\]

\[
= 2500 \left(1 + \frac{1}{600}\right)^{120}
\]

\[
= 2500 \left(\frac{1}{600}\right)^{120}
\]

\[
= 3052.998584656
\]

\[
\approx 3053.00
\]

The final value of the investment is about $3053.00.

48. **COMPUTERS** Zita’s computer is depreciating at a rate of 3% per year. She bought the computer for $1200.

a. Write an equation to represent this situation.

b. What will the computer’s value be after 5 years?

**SOLUTION:**

a. Use the equation for exponential decay, with $a = 1200$ and $r = 0.03$.

\[
y = a(1-r)^t
\]

\[
= 1200(1-0.03)^t
\]

The equation that represents the depreciation of Zita’s computer is $y = 1200(1-0.03)^t$.

b. Substitute 5 for $t$ and solve.

\[
y = 1200(1-0.03)^5
\]

\[
= 1200(0.97)^5
\]

\[
= 1030.48083084
\]

\[
\approx 1030.48
\]

After 5 years, Zita’s computer value is about $1030.48.

**Find the next three terms in each geometric sequence.**

49. $-1, 1, -1, 1, ...$

**SOLUTION:**

Calculate the common ratio.

\[
\begin{array}{cccc}
-1 & 1 & -1 & 1 \\
\frac{-1}{1} & = & -1 & \frac{-1}{1} = -1 \\
\frac{1}{-1} & = & -1 & \frac{1}{-1} = -1 \\
\end{array}
\]

The common ratio is $-1$. Multiply each term by the common ratio to find the next three terms.

$1 \times -1 = -1$

$-1 \times -1 = 1$

$1 \times -1 = -1$

The next three terms of the sequence are $-1, 1,$ and $-1.$
50. 3, 9, 27 ...

**SOLUTION:**
Calculate the common ratio.

\[
\frac{9}{3} = 3 \quad \text{and} \quad \frac{27}{9} = 3
\]

The common ratio is 3. Multiply each term by the common ratio to find the next three terms.

\[
27 \times 3 = 81 \\
81 \times 3 = 243 \\
243 \times 3 = 729
\]

The next three terms of the sequence are 81, 243, and 729.

51. 256, 128, 64, ...

**SOLUTION:**
Calculate the common ratio.

\[
\frac{128}{256} = \frac{1}{2} \quad \text{and} \quad \frac{64}{128} = \frac{1}{2}
\]

The common ratio is \(\frac{1}{2}\). Multiply each term by the common ratio to find the next three terms.

\[
64 \times \frac{1}{2} = 32 \\
32 \times \frac{1}{2} = 16 \\
16 \times \frac{1}{2} = 8
\]

The next three terms of the sequence are 32, 16, and 8.

**Write the equation for the \(n\)th term of each geometric sequence.**

52. \(-1, 1, -1, 1, ...\)

**SOLUTION:**
The first term of the sequence is \(-1\). So, \(a_1 = -1\).

Calculate the common ratio.

\[
\frac{-1}{1} = -1 \\
\frac{-1}{1} = -1 \\
\frac{-1}{1} = -1 \\
\frac{-1}{1} = -1
\]

The common ratio is \(-1\). So, \(r = -1\).

\[
a_n = a_1 \cdot r^{n-1} \\
a_n = -1(-1)^{n-1}
\]
53. 3, 9, 27, ...

**SOLUTION:**
The first term of the sequence is 3. So, \( a_1 = 3 \).
Calculate the common ratio.

\[
\begin{array}{c}
3 \quad 9 \quad 27 \\
\frac{9}{3} = 3 \quad \frac{27}{9} = 3
\end{array}
\]

The common ratio is 3. So, \( r = 3 \).

\[
a_n = a_1 r^{n-1} \\
a_n = 3(3)^{n-1}
\]

54. 256, 128, 64, ...

**SOLUTION:**
The first term of the sequence is 256. So, \( a_1 = 256 \).
Calculate the common ratio.

\[
\begin{array}{c}
256 \quad 128 \quad 64 \\
\frac{128}{256} = \frac{1}{2} \quad \frac{64}{128} = \frac{1}{2}
\end{array}
\]

The common ratio is \( \frac{1}{2} \). So, \( r = \frac{1}{2} \).

\[
a_n = a_1 r^{n-1} \\
a_n = 256 \left( \frac{1}{2} \right)^{n-1}
\]

55. **SPORTS** A basketball is dropped from a height of 20 feet. It bounces to \( \frac{1}{2} \) its height each bounce. Draw a graph to represent the situation.

**SOLUTION:**
Compared to the previous bounce, the ball returns to \( \frac{1}{2} \) its height. So, the common ratio is \( \frac{1}{2} \).
Use common ratio to find next term

\[
a_n = a_1 r^{n-1} \\
a_2 = 20 \left( \frac{1}{2} \right)^{2-1} \\
a_2 = 20 \left( \frac{1}{2} \right)^1 \\
a_2 = 10
\]
The geometric sequence that models this situation is 20, 10, 5, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{5}{16}$, $\frac{5}{32}$, and so forth.

Therefore, the geometric sequence that models this situation is 20, 10, 5, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{5}{16}$, $\frac{5}{32}$, and so forth.
Find the first five terms of each sequence.

56. \( a_1 = 11, a_n = a_{n-1} - 4, n \geq 2 \)

**SOLUTION:**

Use \( a_1 = 11 \) and the recursive formula to find the next four terms.

\[
\begin{align*}
a_2 &= a_{2-1} - 4 & n &= 2 \\
 &= a_1 - 4 & \text{Simplify} \\
 &= 11 - 4 & a_1 &= 11 \\
&= 7 \\

a_3 &= a_{3-1} - 4 & n &= 3 \\
 &= a_2 - 4 & \text{Simplify} \\
 &= 7 - 4 & a_2 &= 7 \\
&= 3 \\

a_4 &= a_{4-1} - 4 & n &= 4 \\
 &= a_3 - 4 & \text{Simplify} \\
 &= 3 - 4 & a_3 &= 3 \\
&= -1 \\

a_5 &= a_{5-1} - 4 & n &= 5 \\
 &= a_4 - 4 & \text{Simplify} \\
&= -1 - 4 & a_4 &= -1 \\
&= -5
\end{align*}
\]

The first five terms are 11, 7, 3, -1, and -5.
57. \(a_1 = 3, a_n = 2a_{n-1} + 6, n \geq 2\)

**SOLUTION:**
Use \(a_1 = 3\) and the recursive formula to find the next four terms.

\[
\begin{align*}
  a_2 &= 2a_1 + 6 & n=2 \\
         &= 2 \cdot 3 + 6 & \text{Simplify} \\
         &= 12 \\
  a_3 &= 2a_2 + 6 & n=3 \\
         &= 2 \cdot 12 + 6 & \text{Simplify} \\
         &= 30 \\
  a_4 &= 2a_3 + 6 & n=4 \\
         &= 2 \cdot 30 + 6 & \text{Simplify} \\
         &= 66 \\
  a_5 &= 2a_4 + 6 & n=5 \\
         &= 2 \cdot 66 + 6 & \text{Simplify} \\
         &= 138
\end{align*}
\]

The first five terms are 3, 12, 30, 66, and 138.

**Write a recursive formula for each sequence.**
58. 2, 7, 12, 17, ...

**SOLUTION:**
Subtract each term from the term that follows it.
\[7 - 2 = 5, 12 - 7 = 5, 17 - 12 = 5\]
There is a common difference of 5. The sequence is arithmetic.
Use the formula for an arithmetic sequence.

\[
\begin{align*}
  a_n &= a_{n-1} + 5 & \text{Recursive formula for arithmetic sequence} \\
  a_1 &= a_{1-1} + 5 & d = 5 \\
  a_1 &= a_0 + 5 \\
\end{align*}
\]

The first term \(a_1\) is 2, and \(n \geq 2\). A recursive formula for the sequence 2, 7, 12, 17, ... is \(a_1 = 2, a_n = a_{n-1} + 5, n \geq 2\).
59. 32, 16, 8, 4, ...

**SOLUTION:**
Subtract each term from the term that follows it.
\[16 - 32 = -16; \ 8 - 16 = -8; \ 4 - 8 = -4\]
There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.
\[\frac{16}{32} = 0.5; \ \frac{8}{16} = 0.5; \ \frac{4}{8} = 0.5\]
There is a common ratio of 0.5. The sequence is geometric.
Use the formula for a geometric sequence.
\[a_n = r a_{n-1} \quad \text{Recursive formula for geometric sequence.}\]
\[a_n = 0.5 a_{n-1} \quad r = 0.5\]
The first term \(a_1\) is 32, and \(n \geq 2\). A recursive formula for the sequence 32, 16, 8, 4, … is \(a_1 = 32, \ a_n = 0.5a_{n-1}, \ n \geq 2\).

60. 2, 5, 11, 23, ...

**SOLUTION:**
Subtract each term from the term that follows it.
\[5 - 2 = 3; \ 11 - 5 = 6; \ 23 - 11 = 12\]
There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.
\[\frac{2}{5} = 0.4; \ \frac{5}{11} \approx 0.45; \ \frac{11}{23} \approx 0.48\]
There is no common ratio. Therefore the sequence must be a combination of both.
From the difference above, you can see each is twice as big as the previous. So \(r = 2\). From the ratios, if each denominator was one less, the ratios would be 0.5. Thus, the common difference is 1. Then, if the first term \(a_1\) is 2, and \(n \geq 2\), a recursive formula for the sequence 2, 5, 11, 23, … is \(a_1 = 2, \ a_n = 2a_{n-1} + 1, \ n \geq 2\).