Simplify each expression. Assume that no denominator equals zero.

1. \( \frac{t^5 u^4}{t^2 u} \)

**SOLUTION:**

\[
\frac{t^5 u^4}{t^2 u} = \left( \frac{t^5}{t^2} \right) \left( \frac{u^4}{u^1} \right) = t^{5-2} u^{4-1} = t^3 u^3
\]

Group powers with the same base. Quotient of Powers Simplify.

2. \( \frac{a^6 b^4 c^{10}}{a^3 b^2 c} \)

**SOLUTION:**

\[
\frac{a^6 b^4 c^{10}}{a^3 b^2 c} = \left( \frac{a^6}{a^3} \right) \left( \frac{b^4}{b^2} \right) \left( \frac{c^{10}}{c} \right) = a^{6-3} b^{4-2} c^{10-1} = a^3 b^2 c^9
\]

Group powers with the same base. Quotient of Powers Simplify.

3. \( \frac{m^8 r^5 p^3}{m^5 r^3 p^3} \)

**SOLUTION:**

\[
\frac{m^8 r^5 p^3}{m^5 r^3 p^3} = \left( \frac{m^8}{m^5} \right) \left( \frac{r^5}{r^3} \right) \left( \frac{p^3}{p^3} \right)
\]

Group powers with the same base. Quotient of Powers Simplify. Zero exponent \( a^0 = 1 \) Simplify.
Simplify each expression. Assume that no denominator equals zero.

4. \( \frac{b^4 c^8 f^8}{b^4 c^3 f^5} \)

**SOLUTION:**

\[
\frac{b^4 c^8 f^8}{b^4 c^3 f^5} = \left( \frac{b^4}{b^4} \right) \left( \frac{c^8}{c^3} \right) \left( \frac{f^8}{f^5} \right)
\]
Group powers with the same base.

\[
= b^{4-4} c^{8-3} f^{8-5}
\]
Quotient of Powers

\[
= b^0 c^5 f^3
\]
Simplify

\[
= c^5 f^3
\]
Zero Exponent \( b^0 = 1 \)

5. \( \frac{g^8 h^2 m}{hg^7} \)

**SOLUTION:**

\[
\frac{g^8 h^2 m}{hg^7} = \left( \frac{g^8}{g^7} \right) \left( \frac{h^2}{h} \right) \left( m \right)
\]
Group powers with the same base.

\[
= g^{8-7} h^{2-1} m
\]
Quotient of Powers

\[
= g^1 h m
\]
Simplify

\[
= gh m
\]
Simplify.

6. \( \frac{r^4 t^7 v^2}{t^7 v^2} \)

**SOLUTION:**

\[
\frac{r^4 t^7 v^2}{t^7 v^2} = \left( \frac{r^4}{1} \right) \left( \frac{t^7}{t^7} \right) \left( \frac{v^2}{v^2} \right)
\]
Group powers with same base.

\[
= r^4 t^{7-7} v^{2-2}
\]
Quotient of Powers

\[
= r^4 t^0 v^0
\]
Simplify

\[
= r^4 (1)(1)
\]
Zero Exponent \( t^0 = 1, v^0 = 1 \)

\[
= r^4
\]
Simplify.
7-2 Division Properties of Exponents

7. \( \frac{x^3y^2z^6}{z^5x^2y} \)

**SOLUTION:**

\[
\frac{x^3y^2z^6}{z^5x^2y} = \left( \frac{x^3}{x^2} \right) \left( \frac{y^2}{y} \right) \left( \frac{z^6}{z^5} \right)
\]

Group powers with same base.

\[
= x^{3-2}y^{2-1}z^{6-5}
\]

Quotient of Powers

\[
= x^1y^1z^1
\]

Simplify.

\[
= xyz
\]

Simplify.

8. \( \frac{n^4q^4w^6}{q^2n^3w} \)

**SOLUTION:**

\[
\frac{n^4q^4w^6}{q^2n^3w} = \left( \frac{n^4}{n^3} \right) \left( \frac{q^4}{q^2} \right) \left( \frac{w^6}{w} \right)
\]

Group powers with same base.

\[
= n^{4-3}q^{4-2}w^{6-1}
\]

Quotient of Powers

\[
= n^1q^2w^5
\]

Simplify.

\[
= nq^2w^5
\]

Simplify.

9. \( \left( \frac{2a^3b^5}{3} \right)^2 \)

**SOLUTION:**

\[
\left( \frac{2a^3b^5}{3} \right)^2 = \frac{(2a^3b^5)^2}{3^2}
\]

Power of a Quotient

\[
= \frac{2^2(a^3)^2(b^5)^2}{9}
\]

Power of a Product

\[
= \frac{4(a^{3\cdot2})(b^{5\cdot2})}{9}
\]

Power of a Power

\[
= \frac{4a^6b^{10}}{9}
\]

Simplify.
10. \( \frac{r^3v^{-2}}{t^{-7}} \)

\textbf{SOLUTION:}

\[
\frac{r^3v^{-2}}{t^{-7}} = \left( \frac{r^3}{1} \right) \left( \frac{v^{-2}}{1} \right) \left( \frac{1}{t^{-7}} \right) \\
= \left( \frac{r^3}{1} \right) \left( \frac{v^{-2}}{1} \right) \left( t^7 \right) \\
= \frac{r^3 t^7}{v^2}
\]

Write as a product of fractions, \( a^{-n} = \frac{1}{a^n} \) and \( \frac{1}{a^{-n}} = a^n \)

Multiply.

11. \( \left( \frac{2c^3d^5}{5g^2} \right)^5 \)

\textbf{SOLUTION:}

\[
\left( \frac{2c^3d^5}{5g^2} \right)^5 = \frac{(2c^3d^5)^5}{(5g^2)^5} \\
= \frac{2^5(c^3)^5(d^5)^5}{5^5(g^2)^5} \\
= \frac{32c^{3\cdot5}d^{5\cdot5}}{3125g^{2\cdot5}} \\
= \frac{32c^{15}d^{25}}{3125g^{10}}
\]

Power of a Quotient, Power of a Product, Power of a Power

Simplify.

12. \( \left( \frac{3xy^2z^4}{x^3yz^4} \right)^0 \)

\textbf{SOLUTION:}

A value to the zero power is 1.

13. \( \left( \frac{3f^4gh^4}{32f^3g^3h} \right)^0 \)

\textbf{SOLUTION:}

A value to the zero power is 1.
7-2 Division Properties of Exponents

14. \( \frac{4r^2y^0t^5}{2rt^3} \)

**SOLUTION:**

\[
\frac{4r^2y^0t^5}{2rt^3} = \left( \frac{4}{2} \right) \left( \frac{r^2}{r} \right) \left( \frac{y^0}{1} \right) \left( \frac{t^5}{t^3} \right)
\]

Group powers with same base

\[= 2r^{2-1}t^{5-3} \quad \text{Quotient of Powers} \]

\[= 2rt^2 \quad \text{Simplify}. \]

15. \( \frac{f^{-3}g^2}{h^{-4}} \)

**SOLUTION:**

\[
\frac{f^{-3}g^2}{h^{-4}} = \left( \frac{f^{-3}}{1} \right) \left( \frac{g^2}{1} \right) \left( \frac{1}{h^{-4}} \right)
\]

Write as a product of fractions

\[= \left( \frac{1}{f^3} \right) \left( \frac{g^2}{1} \right) \left( \frac{h^4}{1} \right) \quad a^{-n} = \frac{1}{a^n} \text{ and } \frac{1}{a^{-n}} = a^n \]

\[= \frac{g^2h^4}{f^3} \quad \text{Multiply}. \]

16. \( \frac{-8x^2y^8z^{-5}}{12x^4y^{-7}z^7} \)

**SOLUTION:**

\[
\frac{-8x^2y^8z^{-5}}{12x^4y^{-7}z^7} = \left( \frac{-8}{12} \right) \left( \frac{x^2}{x^4} \right) \left( \frac{y^8}{y^{-7}} \right) \left( \frac{z^{-5}}{z^7} \right)
\]

Group powers with the same base

\[= \frac{-2}{3}x^{2-4}y^{8-(-7)}z^{-5-7} \quad \text{Quotient of Powers} \]

\[= \frac{-2}{3}x^{-2}y^{15}z^{-12} \quad \text{Simplify}. \]

\[= \frac{-2}{3} \cdot \frac{x^{-2}y^{15}z^{-12}}{1} \quad \text{Write as a product of fractions}. \]

\[= \frac{-2}{3} \cdot \frac{x^{-2}y^{15}z^{-12}}{1} \quad a^{-n} = \frac{1}{a^n} \text{ and } \frac{1}{a^{-n}} = a^n \]

\[= \frac{-2y^{15}}{3x^2z^{12}} \quad \text{Multiply}. \]
17. \[ \frac{2a^2b^{-7}c^{10}}{6a^{-3}b^2c^{-3}} \]

**SOLUTION:**

\[
\frac{2a^2b^{-7}c^{10}}{6a^{-3}b^2c^{-3}} = \left( \frac{2}{6} \right) \left( \frac{a^2}{a^{-3}} \right) \left( \frac{b^{-7}}{b^2} \right) \left( \frac{c^{10}}{c^{-3}} \right) \quad \text{Group powers with the same base}
\]

\[
= \frac{1}{3} a^{2-(-3)} b^{-7-2} c^{10-(-3)} \quad \text{Quotient of Powers}
\]

\[
= \frac{1}{3} a^5 b^{-9} c^{13} \quad \text{Simplify.}
\]

\[
= \frac{1}{3} a^5 \frac{b^{-9}}{b^2} c^{13} \quad \text{Write a a product of fractions}
\]

\[
= \frac{1}{3} a^5 b^{-9-2} c^{13} \quad \text{Multiply.}
\]

\[
= \frac{a^5 c^{13}}{3b^9}
\]

18. **FINANCIAL LITERACY** The gross domestic product (GDP) for the United States in 2008 was $14.204 trillion, and the GDP per person was $47,580. Use order of magnitude to approximate the population of the United States in 2008.

**SOLUTION:**

Since there are 12 zeros in a trillion, the order of magnitude for 14.204 trillion is \(10^{12}\).

Since there are 4 zeros in a ten-thousand, the order of magnitude for 45,780 is \(10^4\).

Divide the GDP for the U.S. by the GDP per person to find the approximate population of the U.S.

\[
\frac{10^{12}}{10^4} = 10^{12-4}
\]

\[
= 10^8
\]

The approximate population of the U.S. in 2008 was \(10^8\) or 100,000,000.

Simplify each expression. Assume that no denominator equals zero.

19. \[ \frac{m^4 p^2}{m^2 p} \]

**SOLUTION:**

\[
\frac{m^4 p^2}{m^2 p} = \left( \frac{m^4}{m^2} \right) \left( \frac{p^2}{p} \right) \quad \text{Group powers with the same base.}
\]

\[
= m^{4-2} p^{2-1} \quad \text{Quotient of Powers}
\]

\[
= m^2 p \quad \text{Simplify.}
\]
7-2 Division Properties of Exponents

20. \( \frac{p^{12}t^3r}{p^2tr} \)

**SOLUTION:**

\[
\frac{p^{12}t^3r}{p^2tr} = \left( \frac{p^{12}}{p^2} \right) \left( \frac{t^3}{t} \right) \left( \frac{r}{r} \right) \\
= p^{12-2}t^{3-1}r^{1-1} \\
= p^{10}t^2 \quad \text{Quotient of Powers} \\
= p^{10}t^2r^0 \quad \text{Simplify.} \\
= p^{10}t^2(1) \\
= p^{10}t^2 \quad \text{Zero Exponent} \ r^0 = 1 \\
= p^{10}t^2 \quad \text{Simplify.}
\]

21. \( \frac{3m^3r^4p^2}{12r^4} \)

**SOLUTION:**

\[
\frac{3m^3r^4p^2}{12r^4} = \frac{3}{12} \frac{m^{-3}r^4p^2}{1} \frac{1}{r^4} \\
= \frac{1}{4} \frac{m^{-3}r^4p^2}{r^4} \frac{1}{1} \\
= \frac{r^4p^2}{4m^3} \quad \text{Multiply}
\]

22. \( \frac{c^4d^4f^3}{c^2d^4f^3} \)

**SOLUTION:**

\[
\frac{c^4d^4f^3}{c^2d^4f^3} = \left( \frac{c^4}{c^2} \right) \left( \frac{d^4}{d^4} \right) \left( \frac{f^3}{f^3} \right) \\
= c^{4-2}d^{4-4}f^{3-3} \\
= c^2d^0f^0 \quad \text{Quotient of Powers} \\
= c^2(1)(1) \quad \text{Simplify.} \\
= c^2 \quad \text{Zero exponent} \ a^0 = 1 \\

= c^2 \quad \text{Simplify.}
\]
23. \( \left( \frac{3xy^4}{5z^2} \right)^2 \)

**SOLUTION:**

\[
\left( \frac{3xy^4}{5z^2} \right)^2 = \frac{(3xy^4)^2}{(5z^2)^2} \quad \text{Power of a Quotient}
\]

\[
= \frac{3^2x^2y^8}{5^2z^4} \quad \text{Power of a Product}
\]

\[
= \frac{3^2x^2y^8}{25z^4} \quad \text{Power of a Power}
\]

\[
= \frac{9x^2y^8}{25z^4} \quad \text{Simplify.}
\]

24. \( \left( \frac{3t^3u^2v^5}{9uv^{21}} \right)^0 \)

**SOLUTION:**

A value to the zero power is 1.

25. \( \left( \frac{p^2t^7}{10} \right)^3 \)

**SOLUTION:**

\[
\left( \frac{p^2t^7}{10} \right)^3 = \left( \frac{p^2t^7}{10} \right)^3 \quad \text{Power of a Quotient}
\]

\[
= \left( \frac{p^2t^7}{10} \right)^3 \quad \text{Power of a Product}
\]

\[
= \frac{p^{2\cdot3}t^{7\cdot3}}{1000} \quad \text{Power of a Power}
\]

\[
= \frac{p^6t^{21}}{1000} \quad \text{Simplify.}
\]
7-2 Division Properties of Exponents

26. \( \frac{x^{-4}y^{9}}{z^{-2}} \)

**SOLUTION:**

\[
\frac{x^{-4}y^{9}}{z^{-2}} = \left( \frac{x^{-4}}{1} \right) \left( \frac{y^{9}}{1} \right) \left( \frac{1}{z^{-2}} \right) \\
= \left( \frac{1}{x^{4}} \right) \left( \frac{y^{9}}{1} \right) \left( \frac{z^{2}}{1} \right) \\
= \frac{y^{9}z^{2}}{x^{4}}.
\]

Write as a product of fractions. Multiply.

27. \( \frac{a^{7}b^{8}c^{8}}{a^{5}bc^{7}} \)

**SOLUTION:**

\[
\frac{a^{7}b^{8}c^{8}}{a^{5}bc^{7}} = \left( \frac{a^{7}}{a^{5}} \right) \left( \frac{b^{8}}{b} \right) \left( \frac{c^{8}}{c^{7}} \right) \\
= a^{7-5}b^{8-1}c^{8-7} \\
= a^{2}b^{7}c^{1} \\
= a^{2}b^{7}c.
\]

Group powers with the same base. Quotient of Powers. Simplify.

28. \( \left( \frac{3np^{3}}{7q^{2}} \right)^{2} \)

**SOLUTION:**

\[
\left( \frac{3np^{3}}{7q^{2}} \right)^{2} = \left( \frac{3np^{3}}{7q^{2}} \right)^{2} \\
= \frac{9n^{2}p^{6}}{49q^{4}}.
\]

29. \( \left( \frac{2r^3t^6}{5u^9} \right)^4 \)

**SOLUTION:**

\[
\left( \frac{2r^3t^6}{5u^9} \right)^4 = \frac{(2r^3t^6)^4}{(5u^9)^4} \\
= \frac{2^4r^{3\cdot4}t^{6\cdot4}}{5^4u^{9\cdot4}} \\
= \frac{16r^{12}t^{24}}{625u^{36}} \\
\]

Power of a Quotient

Power of a Product

Power of a Power

Simplify.

30. \( \left( \frac{3m^5r^3}{4p^8} \right)^4 \)

**SOLUTION:**

\[
\left( \frac{3m^5r^3}{4p^8} \right)^4 = \frac{(3m^5r^3)^4}{(4p^8)^4} \\
= \frac{3^4(m^5)^4(r^3)^4}{4^4(p^8)^4} \\
= \frac{8m^{20}r^{12}}{256p^{32}} \\
\]

Power of a Quotient

Power of a Product

Power of a Power

Simplify.

31. \( \left( \frac{5f^g g^h h^2}{fg^2h^3} \right)^0 \)

**SOLUTION:**

A value to the zero power is 1.
7-2 Division Properties of Exponents

32. Simplify each expression. Assume that no denominator equals zero.

- \( \frac{p^{12} r^2}{p^2 r} \)

**SOLUTION:**

\[
\frac{p^{12} r^2}{p^2 r} = \left( \frac{p^{12}}{p^2} \right) \left( \frac{r^2}{r} \right)
\]

Group powers with the same base

\[
= p^{12-2} r^{2-1}
\]

Quotient of Powers

\[
= p^{10} r^1
\]

Simplify.

\[
= p^{10} r
\]

Zero Exponent \( t^0 = 1 \)

- \( \frac{p^4 r^3}{r^2} \)

**SOLUTION:**

\[
\frac{p^4 r^3}{r^2} = \left( \frac{p^4}{r^2} \right) \left( \frac{r^3}{r^2} \right)
\]

Write as a product of fractions

\[
= \left( \frac{p^4}{r^2} \right) \left( \frac{r^3}{r^2} \right)
\]

\[
= p^{4-2} r^{3-2}
\]

Multiply

34. Simplify each expression. Assume that no denominator equals zero.

- \( -\frac{5c^2 d^5}{8cd^5 f^0} \)

**SOLUTION:**

\[
-\frac{5c^2 d^5}{8cd^5 f^0} = \left( \frac{-5}{8} \right) \left( \frac{c^2}{c} \right) \left( \frac{d^5}{d^5} \right) \left( \frac{1}{f^0} \right)
\]

Group powers with the same base

\[
= \left( \frac{-5}{8} \right) (c^{2-1}) (d^{5-5}) (1)
\]

Quotient of Powers

\[
= -\frac{5}{8} c(1)
\]

Simplify.

\[
= -\frac{5}{8} c(1)(1)
\]

Zero exponent \( d^0 = 1 \)

\[
= -\frac{5c}{8}
\]

Simplify.
35. \[ \frac{-2f^3g^2h^0}{8f^2g^2} \]

**SOLUTION:**

\[ \frac{-2f^3g^2h^0}{8f^2g^2} = \left(\frac{-2}{8}\right) \left(\frac{f^3}{f^2}\right) \left(\frac{g^2}{g^2}\right) \left(\frac{h^0}{1}\right) \]

Group powers with the same base.

\[ = \left(\frac{-2}{8}\right) f^{3-2} g^{2-2} h^0 \]

Quotient of Powers

\[ = \left(\frac{-1}{4}\right) f g^0 1 \]

Simplify.

\[ = \left(\frac{-1}{4}\right) f 1 \]

Zero exponent \( g^0 = 1 \)

\[ = \frac{-f}{4} \]

Simplify.

36. \[ \frac{12m^{-4}p^2}{-15m^3p^{-9}} \]

**SOLUTION:**

\[ \frac{12m^{-4}p^2}{-15m^3p^{-9}} = \left(\frac{12}{-15}\right) \left(\frac{m^{-4}}{m^3}\right) \left(\frac{p^2}{p^{-9}}\right) \]

Group powers with the same base.

\[ = \left(\frac{12}{-15}\right) m^{-4-3} p^{2-(-9)} \]

Quotient of Powers

\[ = \left(\frac{4}{5}\right) m^{-7} p^{11} \]

Simplify.

\[ = \left(\frac{4}{5}\right) \left(\frac{m^{-7}}{1}\right) p^{11} \]

Write as a product of a fraction.

\[ = \left(\frac{4}{5}\right) \left(\frac{1}{m^7}\right) p^{11} \]

\[ a^{-n} = \frac{1}{a^n} \text{ and } \frac{1}{a^{-n}} = a^n \]

\[ = \frac{-4p^{11}}{5m^7} \]

Multiply

37. \[ \frac{k^4m^3p^2}{k^2m^2} \]

**SOLUTION:**

\[ \frac{k^4m^3p^2}{k^2m^2} = \left(\frac{k^4}{k^2}\right) \left(\frac{m^3}{m^2}\right) \left(\frac{p^2}{1}\right) \]

Group powers with the same base

\[ = k^{4-2} m^{3-2} p^2 \]

Quotient of Powers

\[ = k^2mp^2 \]

Simplify.
38. Simplify \( \frac{14f^{-3}g^2h^{-7}}{21k^3} \)

**SOLUTION:**

\[
\frac{14f^{-3}g^2h^{-7}}{21k^3} = \left( \frac{14}{21} \right) \left( \frac{f^{-3}}{1} \right) \left( \frac{g^2}{1} \right) \left( \frac{h^{-7}}{1} \right) \left( \frac{1}{k^3} \right)
\]

Write as a product of fractions

\[
= \frac{2}{3} \left( \frac{1}{f^3} \right) \left( \frac{1}{g^2} \right) \left( \frac{1}{h^7} \right) \left( \frac{1}{k^3} \right)
\]

\[
= \frac{2g^2}{3f^3h^7k^3}
\]

Multiply

39. Simplify \( \frac{39t^4uv^{-2}}{13t^{-3}u^7} \)

**SOLUTION:**

\[
\frac{39t^4uv^{-2}}{13t^{-3}u^7} = \left( \frac{39}{13} \right) \left( \frac{t^4}{t^{-3}} \right) \left( \frac{u}{u^7} \right) \left( \frac{v^{-2}}{1} \right)
\]

Group powers with the same base

\[
= 3t^{4-(-3)}u^{1-7}v^{-2}
\]

Quotient of Products

\[
= 3t^7u^{-6}v^{-2}
\]

Simplify

\[
= 3t^7 \frac{1}{u^6v^2}
\]

Write as a product of fractions

\[
a^{-n} = \frac{1}{a^n} \quad \text{and} \quad \frac{1}{a^{-n}} = a^n
\]

Multiply
7-2 Division Properties of Exponents

40. \( \left( \frac{a^{-2}b^4c^5}{a^3b^2c^3} \right)^2 \)

**SOLUTION:**

\[
\left( \frac{a^{-2}b^4c^5}{a^3b^2c^3} \right)^2 = \frac{(a^{-2}b^4c^5)^2}{(a^3b^2c^3)^2} \]

Power of a Quotient

\[
= \frac{(a^{-2})^2(b^4)^2(c^5)^2}{(a^3)^2(b^2)^2(c^3)^2} \]

Power of a Product

\[
= \frac{a^{-4}b^8c^{10}}{a^6b^4c^6} \]

Power of a Power

\[
= a^{-4+8}b^{8+10}c^{10-6} \]

Quotient of Powers

\[
= a^4b^4c^4 \]

Simplify.

41. \( \frac{r^3t^{-1}x^{-5}}{tx^5} \)

**SOLUTION:**

\[
\frac{r^3t^{-1}x^{-5}}{tx^5} = \left( \frac{r^3}{t} \right) \left( \frac{t^{-1}}{t} \right) \left( \frac{x^{-5}}{x^5} \right) \]

Group powers with the same base

\[
= r^{3-1}t^{-1-1}x^{-5-5} \]

Quotient of Powers

\[
= r^2t^{-2}x^{-10} \]

Simplify.

\[
= \frac{r^3}{t^2} \]

Write as a product of fractions.

\[
a^{-n} = \frac{1}{a^n} \text{ and } \frac{1}{a^{-n}} = a^n \]

Multiply
7-2 Division Properties of Exponents

42. \( \frac{g^6 h^7 j^2}{g^{-5} h^0 j^{-2}} \)

**SOLUTION:**

\[
\frac{g^6 h^7 j^{-2}}{g^{-5} h^0 j^{-2}} = \left( \frac{g^6}{g^{-5}} \right) \left( \frac{h^7}{h^0} \right) \left( \frac{j^{-2}}{j^{-2}} \right)
\]

Group powers with the same base.

\[
g^6 \cdot (-5) h^7 - 0 j^{-2} - (-2) = g^5 h^7 j^0
\]

Quotient of Powers

\[
g^5 h^7 (1) = g^5 h^7
\]

Zero Exponent

\[
g^5 h^7
\]

Simplify.

43. **INTERNET** In a recent year, there were approximately 3.95 million Internet hosts. Suppose there were 208 million Internet users. Determine the order of magnitude for the Internet hosts and Internet users. Using the orders of magnitude, how many Internet users were there compared to Internet hosts?

**SOLUTION:**

Since there are 6 zeros in a million, the order of magnitude for 3.95 million is: \(10^6\). Since there are 8 zeros in a hundred-million, the order of magnitude for 208 million is \(10^8\). The numbers differ by an order of magnitude of \(10^2\), so there were 100 times as many Internet users as Internet hosts.

44. **PROBABILITY** The probability of rolling a die and getting an even number is \(\frac{1}{2}\). If you roll the die twice, the probability of getting an even number twice is \(\left( \frac{1}{2} \right) \left( \frac{1}{2} \right)\) or \(\left( \frac{1}{2} \right)^2\).

a. What does \(\left( \frac{1}{2} \right)^4\) represent?

b. Write an expression to represent the probability of rolling a die \(d\) times and getting an even number every time. Write the expression as a power of 2.

**SOLUTION:**

a. probability of all evens on 4 rolls

b.

\[
\left( \frac{1}{2} \right)^d;
\]

\[
\left( \frac{1}{2} \right)^d = \frac{1^d}{2^d} \quad \text{Power of a Quotient.}
\]

\[
= \frac{1}{2^d} \quad \text{Simplify.}
\]

\[
= 2^{-d} \quad \frac{1}{a^n} = a^{-n}
\]
Simplify each expression. Assume that no denominator equals zero.

45. \( \frac{-4w^{12}}{12w^3} \)

SOLUTION:
\[
\frac{-4w^{12}}{12w^3} = \left( \frac{-4}{12} \right) \left( \frac{w^{12}}{w^3} \right) \quad \text{Group powers with the same base.}
\]
\[
= \frac{-1}{3}w^{12-3} \quad \text{Quotient of Powers}
\]
\[
= -\frac{w^9}{3} \quad \text{Simplify.}
\]

46. \( \frac{13r^7}{39r^4} \)

SOLUTION:
\[
\frac{13r^7}{39r^4} = \left( \frac{13}{39} \right) \left( \frac{r^7}{r^4} \right) \quad \text{Group powers with the same base}
\]
\[
= \frac{1}{3}r^{7-4} \quad \text{Quotient of Powers}
\]
\[
= \frac{r^3}{3} \quad \text{Simplify.}
\]
47. \( \frac{(4k^3 m^2)^3}{(5k^2 m^{-3})^{-2}} \)

**SOLUTION:**

\[
\frac{(4k^3 m^2)^3}{(5k^2 m^{-3})^{-2}} = \frac{4^3 (k^3)^3 (m^2)^3}{5^{-2} (k^2)^{-2} (m^{-3})^{-2}} \\
= \frac{4^{3 \cdot 3} k^{3 \cdot 3} m^{2 \cdot 3}}{5^{-2} k^{2 \cdot -2} m^{-3 \cdot -2}} \\
= \frac{64k^9 m^6}{(25)^{-1} k^{-4} m^6} \\
= \left( \frac{64}{25} \right) k^{9 - (-4)} m^{6 - 6} \\
= (64)(25)k^{13}m^0 \\
= 1600k^{13} \cdot (1) \\
= 1600k^{13}.
\]

Power of Products

Power of Powers

Simplify

Group powers with the same base.

Quotient of Powers

Simplify.

Zero Exponent

Simplify.
Simplify each expression. Assume that no denominator equals zero.

48. \( \frac{3w^2}{y} \)

SOLUTION:

\[
\frac{3w^2}{y} = \frac{3w^2}{y} \quad \text{Power of a Product}
\]

\[
= \frac{3w^2}{y} \quad \text{Power of a Power}
\]

\[
= \frac{3w^2}{y} \quad \text{Simplify.}
\]

\[
= \left( \frac{3}{1} \right) \left( \frac{w}{w^{-3}} \right) \left( \frac{y^{-2}}{y^3} \right) \quad \text{Group powers with the same base.}
\]

\[
= 3w^{1-(-3)}y^{-2-3} \quad \text{Quotient of Powers}
\]

\[
= 3w^4y^{-5}
\]

\[
= \frac{3w^4}{y^5} \quad \text{Write as a product of fractions.}
\]

\[
= \frac{3w^4}{y^5} \quad \text{Multiply}
\]

49. \( \frac{20gr^2t^{-5}}{4q^0r^4t^{-2}} \)

SOLUTION:

\[
\frac{20gr^2t^{-5}}{4q^0r^4t^{-2}} = \left( \frac{20}{4} \right) \left( \frac{q^1}{q^0} \right) \left( \frac{r^{2-4}}{t^{-2-(-5)}} \right) \quad \text{Group powers with the same base.}
\]

\[
= 5q^{1-0}r^{-2-4}t^{-5-(-2)} \quad \text{Quotient of Powers}
\]

\[
= 5q^1r^{-6}t^{-3} \quad \text{Simplify.}
\]

\[
= 5q^1r^{-6}t^{-3} \quad \text{Write as a product of fractions.}
\]

\[
= 5q^{1}r^{-6}t^{-3} \quad \text{Multiply}
\]

\[
= \frac{5q}{r^6t^3}
\]
Solve each expression. Assume that no denominator equals zero.

50. \[ \frac{-12c^3d^9f^{-2}}{6c^5d^{-3}f^4} \]

**SOLUTION:**

\[
\frac{-12c^3d^0f^{-2}}{6c^5d^{-3}f^4} = \left( \frac{-12}{6} \right) \left( \frac{c^3}{c^5} \right) \left( \frac{d^0}{d^{-3}} \right) \left( \frac{f^{-2}}{f^4} \right)
\]

Group powers with the same base.

\[
= -2c^{3-5}d^{0-(-3)}f^{-2-4}
\]

Quotient of powers

\[
= -2c^{-2}d^3f^{-6}
\]

Simplify.

\[
= -\frac{2c^{-2}}{d^3f^6}
\]

Write as a product of fractions

\[
= -\frac{2d^3}{c^2f^6}
\]

Simplify.
51. \[ \frac{(2g^3h^{-2})^2}{(g^2h^0)^3} \]

**SOLUTION:**

\[
\frac{(2g^3h^{-2})^2}{(g^2h^0)^3} = \frac{2^2(g^3)^2(h^{-2})^2}{(g^2)^3(h^0)^3} \quad \text{Power of a Product}
\]

\[
= \frac{2^2g^{3\cdot2}h^{-2\cdot2}}{g^{2\cdot3}h^{0\cdot3}} \quad \text{Power of a Power}
\]

\[
= \frac{4g^6h^{-4}}{g^6h^0} \quad \text{Simplify.}
\]

\[
= 4\left(\frac{g^6}{g^6}\right)\left(h^{-4}/h^0\right) \quad \text{Group powers with same base.}
\]

\[
= 4g^{6-(-6)}h^{-4-0} \quad \text{Quotient of Powers}
\]

\[
= 4g^{12}h^{-4} \quad \text{Simplify.}
\]

\[
= 4g^{12}h^{-4}/1 \quad \text{Write as a product of fractions.}
\]

\[
= 4g^{12}/h^4 \quad a^{-n} = \frac{1}{a^n} \text{and} \frac{1}{a^{-n}} = a^n
\]

\[
= \frac{4g^{12}}{h^4} \quad \text{Simplify.}
\]
Simplify each expression. Assume that no denominator equals zero.

52. \( \frac{(5 p r^{-2})^2}{(3 p^{-1} r)^3} \)

**SOLUTION:**

\[
\frac{(5 p r^{-2})^2}{(3 p^{-1} r)^3} = \frac{1}{(3 p^{-1} r)^3} \cdot \frac{5^2 p^2 r^{-2}}{1} \\
= \frac{1}{(3 p^{-1} r)^3} \cdot \frac{25 p^2 r^{-2}}{1} \\
= \frac{1}{(27 p^{-1} r)^3} \cdot \frac{25 p^2 r^{-2}}{1} \\
= \frac{1}{675 p^{-1} r^{-1}} \cdot \frac{25 p^2 r^{-2}}{1} \\
= \frac{pr}{675} \\
= \frac{1}{a^{-n}} = a^n
\]

\( a^{-n} = \frac{1}{a^n} \) 

Power of Products

Power of Powers

Group powers with the same base.

Product of Powers

Simplify.

\( \frac{1}{a^{-n}} = a^n \)
53. \[ \left( \frac{-3x^{-6}y^{-1}z^{-2}}{6x^{-2}yz^{-5}} \right)^{-2} \]

**SOLUTION:**

\[
\left( \frac{-3x^{-6}y^{-1}z^{-2}}{6x^{-2}yz^{-5}} \right)^{-2} = \left( \frac{-3x^{-6}y^{-1}z^{-2}}{6x^{-2}yz^{-5}} \right)^{-2} \\
= \left( \frac{6x^{-2}yz^{-5}}{-3x^{-6}y^{-1}z^{-2}} \right)^{2} \\
= \frac{6^2(x^{-2})^2(y)^2(z^{-5})^2}{(-3)^2(x^{-6})^2(y^{-1})^2(z^{-2})^2} \\
= \frac{6^2x^{-2}y^2z^{-5}2}{(-3)^2x^{-6}y^{-1}z^{-2}2} \\
= \frac{6^2x^{-4}y^{-4}z^{-10}}{-3^2x^{-12}y^{-2}z^{-4}} \\
= \left( \frac{6}{9} \right) x^{-4+12}y^4z^{-10+4} \\
= \left( \frac{36}{9} \right) x^8y^4z^{-6} \\
= 4x^8y^4z^{-6} \\
= \frac{4x^8y^4}{z^6} \\
= 4x^8y^4z^{-6} \\
= \frac{1}{a^{-n}} = a^n \\
\]

Power of a Quotient

\[ a^{-n} = \frac{1}{a^n} \text{ and } \frac{1}{a^{-n}} = a^n \]

Power of a Product

Simplify.

Group powers with the same base.

Quotient of Powers

Simplify.

Write as a product of fractions.

Simplify.
7-2 Division Properties of Exponents

54. \( \left( \frac{2a^2b^4c^2}{-4a^2b^5c^{-7}} \right)^{-1} \)

\textbf{SOLUTION:}

\[
\left( \frac{2a^{-2}b^{4}c^{2}}{-4a^{-2}b^{5}c^{-7}} \right)^{-1} = \frac{\left(2a^{-2}b^{4}c^{2}\right)^{-1}}{\left(-4a^{-2}b^{5}c^{-7}\right)^{-1}}
\]

\[
= \frac{-4a^{-2}b^{5}c^{-7}}{2a^{-2}b^{4}c^{2}}
\]

\[
= \left( \frac{-4}{2} \right) \left( \frac{a^{-2}}{a^{-2}} \right) \left( \frac{b^{5}}{b^{4}} \right) \left( \frac{c^{-7}}{c^{2}} \right)
\]

\[
= \frac{-2a^{-2+2}b^{5-4}c^{-7-2}}{1}
\]

\[
= -2 \frac{a^{0}b^{-9}c^{-9}}{1}
\]

\[
= -2 \frac{\frac{1}{b^{9}} \frac{1}{c^{9}}}{1}
\]

\[
= -2 \frac{1}{b^{9}c^{9}}
\]

\[
= \frac{2}{b^{9}c^{9}}
\]

\[
\text{Power of a Quotient}
\]

\[
a^{-n} = \frac{1}{a^n} \text{ and } -\frac{1}{a^{-n}} = a^n
\]

55. \( \left( \frac{16x^{2}y^{-1}}{4x^{0}y^{-4}z^{2}} \right)^{0} \)

\textbf{SOLUTION:}

\[
\frac{\left(16x^{2}y^{-1}\right)^{0}}{\left(4x^{0}y^{-4}z^{2}\right)^{-2}} = \frac{1}{\left(4x^{0}y^{-4}z^{2}\right)^{-2}}
\]

\[
= \left(4x^{0}y^{-4}z^{2}\right)^{2}
\]

\[
= 4^{2} (x^{0})^{2} (y^{-4})^{2} z^{2}
\]

\[
= 16x^{0}y^{-8}z^{4}
\]

\[
= 16 \frac{y^{-8}}{z^{4}}
\]

\[
= 16 \frac{1}{y^{8}z^{2}}
\]

\[
= \frac{1}{y^{8}z^{2}}
\]

\[
\text{Zero Exponent}
\]

\[
a^{-n} = \frac{1}{a^n}
\]

\[
\text{Power of a Product}
\]

\[
\text{Power of a Power}
\]

\[
\text{Simplify.}
\]

\[
\text{Write as a product of fractions}
\]

\[
\frac{1}{y^{8}z^{2}}
\]

\[
\text{Simplify.}
\]
Simplify each expression. Assume that no denominator equals zero.

1. SOLUTION: 

2. SOLUTION: 

3. SOLUTION: 

4. SOLUTION: 

5. SOLUTION: 

6. SOLUTION: 

7. SOLUTION: 

8. SOLUTION: 

9. SOLUTION: 

10. SOLUTION: 

11. SOLUTION: 

12. SOLUTION: 

13. SOLUTION: 

14. SOLUTION: 

15. SOLUTION: 

16. SOLUTION: 

17. SOLUTION: 

18. SOLUTION: 

19. SOLUTION: 

20. SOLUTION: 

21. SOLUTION: 

22. SOLUTION: 

23. SOLUTION: 

24. SOLUTION: 

25. SOLUTION: 

26. SOLUTION: 

27. SOLUTION: 

28. SOLUTION: 

29. SOLUTION: 

30. SOLUTION: 

31. SOLUTION: 

32. SOLUTION: 

33. SOLUTION: 

34. SOLUTION: 

35. SOLUTION: 

36. SOLUTION: 

37. SOLUTION: 

38. SOLUTION: 

39. SOLUTION: 

40. SOLUTION: 

41. SOLUTION: 

42. SOLUTION: 

43. SOLUTION: 

44. SOLUTION: 

45. SOLUTION: 

46. SOLUTION: 

47. SOLUTION: 

48. SOLUTION: 

49. SOLUTION: 

50. SOLUTION: 

51. SOLUTION: 

52. SOLUTION: 

53. SOLUTION: 

54. SOLUTION: 

55. SOLUTION: 

56. SOLUTION: 

57. CCSS SENSE-MAKING The processing speed of an older computer is about $10^8$ instructions per second. A new computer can process about $10^{10}$ instructions per second. The newer computer is how many times as fast as the older one? 

SOLUTION: 

$10^{10}$ and $10^8$ differ by an order of magnitude of $10^2$, so the newer computer is 100 times faster than the older computer.
7-2 Division Properties of Exponents

58. ASTRONOMY The brightness of a star is measured in magnitudes. The lower the magnitude, the brighter the star.

A magnitude 9 star is 2.51 times as bright as a magnitude 10 star. A magnitude 8 star is 2.51 \cdot 2.51 or 2.51^2 times as bright as a magnitude 10 star.

a. How many times as bright is a magnitude 3 star as a magnitude 10 star?
b. Write an expression to compare a magnitude \( m \) star to a magnitude 10 star.
c. A full moon is considered to be approximately magnitude –13. Does your expression make sense for this magnitude? Explain.

SOLUTION:

b. A magnitude \( m \) star would be \[ \frac{2.51^{10}}{2.51^m} = 2.51^{10-m} \] times as bright as a magnitude 10 star.

c. According to the expression, a full Moon would be \[ 2.51^{10-(-13)} = 2.51^{23} \] or 1,557,742,231 times as bright as a magnitude 10 star. Since we know that the lower the magnitude the brighter the object, it follows that a magnitude -13 object is significantly brighter than a magnitude 10 object. The expression in part \( b \) does make sense.

59. PROBABILITY The probability of rolling a die and getting a 3 is \( \frac{1}{6} \). If you roll the die twice, the probability of getting a 3 both times is \( \frac{1}{6} \cdot \frac{1}{6} \) or \( \left( \frac{1}{6} \right)^2 \).

a. Write an expression to represent the probability of rolling a die \( d \) times and getting a 3 each time.
b. Write the expression as a power of 6.

SOLUTION:

b. \[ \left( \frac{1}{6} \right)^d = \frac{1^d}{6^d} = (1)6^{-d} = 6^{-d} \]

60. MULTIPLE REPRESENTATIONS To find the area of a circle, use \( A = \pi r^2 \). The formula for the area of a square is \( A = s^2 \).
7-2 Division Properties of Exponents

![Diagram of a circle with radius r]

a. **ALGEBRAIC** Find the ratio of the area of the circle to the area of the square.

b. **ALGEBRAIC** If the radius of the circle and the length of each side of the square is doubled, find the ratio of the area of the circle to the square.

c. **TABULAR** Copy and complete the table.

<table>
<thead>
<tr>
<th>Radius</th>
<th>Area of Circle</th>
<th>Area of Square</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2r</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3r</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4r</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5r</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6r</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. **ANALYTICAL** What conclusion can be drawn from this?

**SOLUTION:**

a. Use the diagram to write an equation for $s$ in terms of $r$ to substitute into the equation for the area of a square. The value of $s$ is the length of one side of the square, which is equal to the diameter of the circle. Therefore, $s = 2r$.

\[
\frac{\text{area of circle}}{\text{area of square}} = \frac{\pi r^2}{(2r)^2}
\]

\[
\frac{\pi r^2}{(2r)^2} = \frac{\pi r^2}{2^2 r^2}
\]

\[
= \pi (\frac{1}{4})^2
\]

\[
= \pi (\frac{1}{4})^0
\]

\[
= \pi (\frac{1}{4})(1)
\]

\[
= \frac{\pi}{4}
\]

b. Replace $r$ with $2r$ in the equations.

\[
s = 2 \cdot 2r = 4r
\]
7-2 Division Properties of Exponents

\[
\frac{\pi(2r)^2}{(4r)^2} = \frac{\pi2^2r^2}{4^2r^2} = \pi\left(\frac{4}{16}\right)r^{2-2} = \pi\left(\frac{1}{4}\right)r^0 = \pi\left(\frac{1}{4}\right)(1) = \frac{\pi}{4}
\]

c.

<table>
<thead>
<tr>
<th>Radius</th>
<th>Area of Circle</th>
<th>Area of Square</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r)</td>
<td>(\pi r^2)</td>
<td>(4r^2)</td>
<td>(\frac{\pi}{4})</td>
</tr>
<tr>
<td>(2r)</td>
<td>(\pi 4r^2)</td>
<td>(16r^2)</td>
<td>(\frac{\pi}{4})</td>
</tr>
<tr>
<td>(3r)</td>
<td>(\pi 9r^2)</td>
<td>(36r^2)</td>
<td>(\frac{\pi}{4})</td>
</tr>
<tr>
<td>(4r)</td>
<td>(\pi 16r^2)</td>
<td>(64r^2)</td>
<td>(\frac{\pi}{4})</td>
</tr>
<tr>
<td>(5r)</td>
<td>(\pi 25r^2)</td>
<td>(100r^2)</td>
<td>(\frac{\pi}{4})</td>
</tr>
<tr>
<td>(6r)</td>
<td>(\pi 36r^2)</td>
<td>(144r^2)</td>
<td>(\frac{\pi}{4})</td>
</tr>
</tbody>
</table>

d. The ratio of the area of the circle to the area of the square will always be \(\frac{\pi}{4}\).

61. **Reasoning** Is \(x^y \cdot x^z = x^{y+z}\) sometimes, always, or never true? Explain.

**Solution:**
Sometimes; sample answer: The equation is true whenever \(x=1\). The equation is false when \(x=2, y=3, \) and \(z=4\).
7-2 Division Properties of Exponents

62. OPEN ENDED Name two monomials with a quotient of \(24a^2b^3\).

SOLUTION:
You need to find two monomials, I and II, such that the terms in monomial I divided by the corresponding terms in monomial II equal \(24a^2b^3\).

The easiest way to do this is to first create monomial II using all of the available terms.

Monomial II = \(a^2b^3\).

Now, multiply this monomial by \(24a^2b^3\) and the result will be monomial I.

\[a^2b^3 \times 24a^2b^3 = 24a^4b^6\]

Check your answer.

\[
\frac{24a^4b^6}{a^2b^3} = 24 \left(\frac{a^4}{a^2}\right) \left(\frac{b^6}{b^3}\right)
= 24a^{4-2}b^{6-3}
= 24a^2b^3
\]

63. CHALLENGE Use the Quotient of Powers Property to explain why \(x^{-n} = \frac{1}{x^n}\).

SOLUTION:
\[
\frac{1}{x^n} = \frac{x^0}{x^n} \quad \text{Zero Exponent } a^0 = 1
= x^{0-n} \quad \text{Quotient of Powers}
= x^{-n} \quad \text{Simplify.}
\]

64. CCSS REGULARITY Write a convincing argument to show why \(3^0 = 1\).

SOLUTION:
We can use the Quotient of Powers Property to show that \(3^0 = 1\).

\[
3^5 = 243
\]
\[
\frac{3^5}{3^5} = \frac{243}{3^5} \quad \text{Divide by } 3^5.
\]
\[
3^{5-5} = \frac{243}{3^5} \quad \text{Quotient of powers.}
\]
\[
3^0 = \frac{243}{243} \quad \text{Simplify.}
\]
\[
3^0 = 1 \quad \text{Simplify.}
\]
65. **WRITING IN MATH**  Explain how to use the Quotient of Powers property and the Power of a Quotient property.

**SOLUTION:**

The Quotient of Powers Property is used when dividing two powers with the same base. The exponents are subtracted. Consider the following example of the Quotient of Powers Property.

\[ \frac{2x^4 y^6 z^4}{5x^2 y^3 z^2} = \frac{2x^4 y^6 z^4}{5x^2 y^3 z^2} \]

Group powers with the same base.

\[ = \frac{2x^4}{5x^2} \frac{y^6}{y^3} \frac{z^4}{z^2} \quad \text{Quotient of Powers} \]

\[ = \frac{2}{5} x^{4-2} y^{6-3} z^{4-2} \quad \text{Simplify} \]

The Power of a Quotient Property is used to find the power of a quotient. You find the power of the numerator and the power of the denominator. Consider the following example of the Power of a Quotient Property.

\[ \left( \frac{2x^4 y^6}{5x^2 y^3} \right)^2 = \left( \frac{2x^4 y^6}{5x^2 y^3} \right)^2 \]

Power of a Quotient.

\[ = \frac{(2x^4)^2 (y^6)^2}{(5x^2)^2 (y^3)^2} \]

Power of Powers

\[ = \frac{2^2 (x^4)^2 (y^6)^2}{5^2 (x^2)^2 (y^3)^2} \]

\[ = \frac{4x^8 y^{12}}{25x^4 y^6} \quad \text{Simplify} \]
66. What is the perimeter of the figure in meters?

No values are provided for horizontal sides \( a, b \) and \( c \). However, the sum of \( a, b, \) and \( c \) is \( 20x \). There are also no values provided for the vertical sides \( d \) and \( e \). However, the sum of \( d \) and \( e \) is \( 12x + 8x \) or \( 20x \).

\[
P = (\text{length of known sides}) + (\text{length of horizontal unknown sides}) + (\text{length of vertical unknowns})
\]

\[
= (12x + 8x + 20x) + 20x + 20x
\]

\[
= 40x + 20x + 20x
\]

\[
= 80x
\]

The correct answer is B.
7-2 Division Properties of Exponents

67. In researching her science project, Leigh learned that light travels at a constant rate and that it takes 500 seconds for light to travel the 93 million miles from the Sun to Earth. Mars is 142 million miles from the Sun. About how many seconds will it take for light to travel from the Sun to Mars?
   F  235 seconds
   G  327 seconds
   H  642 seconds
   J  763 seconds

SOLUTION:
Set up a proportion of seconds to millions of miles.

\[
\frac{\text{seconds to Earth}}{500} = \frac{\text{seconds to Mars}}{93}
\]

\[
\frac{93x}{142} = 71,000
\]

The correct answer is J.

68. EXTENDED RESPONSE  Jessie and Jonas are playing a game using the spinners shown. Each spinner is equally likely to stop on any of the four numbers. In the game, a player spins both spinners and calculates the product of the two numbers on which the spinners have stopped.

a. What product has the greatest probability of occurring?
b. What is the probability of that product occurring?

SOLUTION:
a. The possible products are 1, 2, 3, 4, 6, 8, 9, 12, and 16. The only product that can be made by two different sets of numbers is 4 (2 • 2 and 1 • 4). So, 4 is the product with the greatest probability of occurring.
b. The number of possible combinations is 4 • 4, or 16. There are three possible spins that would produce a product of 4: 1, 4; 4, 1; 2, 2.

The probability of the product of 4 occurring is \(\frac{3}{16}\).
7-2 Division Properties of Exponents

69. Simplify \((4^{-2} \cdot 5^0 \cdot 64)^3\).

A \(\frac{1}{64}\)  
B 64  
C 320  
D 1024

**SOLUTION:**
\[
(4^{-2} \cdot 5^0 \cdot 64)^3 = \left(\frac{1}{16} \cdot 1 \cdot 64\right)^3
\]
\[
= (\frac{64}{16})^3
\]
\[
= 4^3
\]
\[
= 64
\]
The correct answer is B.

70. GEOMETRY A rectangular prism has a width of \(7x^3\) units, a length of \(4x^2\) units, and a height of \(3x\) units. What is the volume of the prism?

**SOLUTION:**
\[
V = \ell \cdot w \cdot h \\
= (7x^3)(4x^2)(3x) \quad \text{Volume formula for a prism.}
\]
\[
= 84x^6 \quad \text{Substitution.}
\]
\[
= 84x^6 \quad \text{Product of Powers}
\]
\[
= 84x^6 \quad \text{Simplify.}
\]

Solve each system of inequalities by graphing.

71. \(y \geq 1\)  
\(x < -1\)

**SOLUTION:**
Graph each inequality. The graph of \(y \geq 1\) is solid and is included in the graph of the solution. The graph of \(x < -1\) is dashed and is not included in the graph of the solution. The solution of the system is the set of ordered pairs in the intersection of the graphs of \(y \geq 1\) and \(x < -1\). This region is darkly shaded in the graph below.
7-2 Division Properties of Exponents

72. \( y \geq -3 \)
\( y - x < 1 \)

**SOLUTION:**
Rewire inequality 2 in slope-intercept form.
\[
y - x < 1 \quad \text{Original inequality 2}
\]
\[
y - x + x < x + 1 \quad \text{Add } x \text{ to each side.}
\]
\[
y < x + 1 \quad \text{Simplify.}
\]
Graph each inequality. The graph of \( y \geq -3 \) is solid and is included in the graph of the solution. The graph of \( y - x < 1 \) is dashed and is not included in the graph of the solution. The solution of the system is the set of ordered pairs in the intersection of the graphs of \( y \geq -3 \) and \( y - x < 1 \). This region is darkly shaded in the graph below.

![Graph of inequality](image1.png)

73. \( y < 3x + 2 \)
\( y \geq -2x + 4 \)

**SOLUTION:**
Graph each inequality. The graph of \( y < 3x + 2 \) is dashed and is not included in the graph of the solution. The graph of \( y \geq -2x + 4 \) is solid and is included in the graph of the solution. The solution of the system is the set of ordered pairs in the intersection of the graphs of \( y < 3x + 2 \) and \( y \geq -2x + 4 \). This region is darkly shaded in the graph below.

![Graph of inequality](image2.png)
7-2 Division Properties of Exponents

74. \( y - 2x < 2 \)
\( y - 2x > 4 \)

**SOLUTION:**
Rewrite each inequality in slope-intercept form first.

**Equation 1:**
\[
\begin{align*}
    y - 2x &< 2 & \text{Original inequality 1} \\
    y - 2x + 2 &< 2x + 2 & \text{Add } 2x \text{ to each side.} \\
    y &< 2x + 2 & \text{Simplify.}
\end{align*}
\]

**Equation 2:**
\[
\begin{align*}
    y - 2x &> 4 & \text{Original inequality 2} \\
    y - 2x + 2 &> 2x + 4 & \text{Add } 2x \text{ to each side.} \\
    y &> 2x + 4 & \text{Simplify.}
\end{align*}
\]

Graph each inequality. The graph of \( y - 2x < 2 \) is dashed and is not included in the graph of the solution. The graph of \( y - 2x > 4 \) is also dashed and is not included in the graph of the solution. The solution of the system is the set of ordered pairs in the intersection of the graphs of \( y - 2x < 2 \) and \( y - 2x > 4 \). There is no shared area, so there is no solution.
Solve each inequality. Check your solution.

75. \(5(2h - 6) > 4h\)

**SOLUTION:**
\[
\begin{align*}
5(2h - 6) &> 4h \\
10h - 30 &> 4h \\
10h - 4h - 30 &> 4h - 4h \\
6h - 30 &> 0 \\
6h - 30 + 30 &> 0 + 30 \\
6h &> 30 \\
\frac{6h}{6} &> \frac{30}{6} \\
h &> 5
\end{align*}
\]
To check, substitute a value greater than 5 for \(h\) in the inequality.

\[
5 [2(6) - 6)] > 4(6)
\]
\[
5(12 - 6) > 24
\]
\[
5(6) > 24
\]
\[
30 > 24
\]
The solution checks.

76. \(22 \geq 4(b - 8) + 10\)

**SOLUTION:**
\[
\begin{align*}
22 &\geq 4(b - 8) + 10 \\
22 &\geq 4b - 32 + 10 \\
22 &\geq 4b - 22 \\
22 + 22 &\geq 4b - 22 + 22 \\
44 &\geq 4b \\
\frac{44}{4} &\geq \frac{4b}{4} \\
11 &\geq b \\
b &\leq 11
\end{align*}
\]
To check, substitute a value less than or equal to 11 for \(b\) in the inequality.

\[
22 \geq 4(b - 8) + 10
\]
\[
22 \geq 4(11 - 8) + 10
\]
\[
22 \geq 4(3) - 10
\]
\[
22 \geq 12 - 10
\]
\[
22 \geq 2
\]
The solution checks.
7-2 Division Properties of Exponents

77. \(5(u - 8) \leq 3(u + 10)\)

**SOLUTION:**
\[
\begin{align*}
5(u - 8) & \leq 3(u + 10) \\
5u - 40 & \leq 3u + 30 \\
5u - 3u & \leq 3u - 3u + 30 \\
2u & \leq 70 \\
\frac{2u}{2} & \leq \frac{70}{2} \\
u & \leq 35 \\
\end{align*}
\]
To check, substitute a value less than or equal to 35 for \(u\) in the inequality.
\(5(u - 8) \leq 3(u + 10)\)
\[
\begin{align*}
5(9 - 8) & \leq 3(9 + 10) \\
5(1) & \leq 3(19) \\
5 & \leq 95 \\
\end{align*}
\]
The solution checks.

78. \(8 + t \leq 3(t + 4) + 2\)

**SOLUTION:**
\[
\begin{align*}
8 + t & \leq 3(t + 4) + 2 \\
8 + t & \leq 3t + 12 + 2 \\
8 + t & \leq 3t + 14 \\
8 - 8 + t & \leq 3t + 14 - 8 \\
t & \leq 3t + 6 \\
t - 3t & \leq 3t - 3t + 6 \\
-2t & \leq 6 \\
\frac{-2t}{-2} & \geq \frac{6}{-2} \\
t & \geq -3 \\
\end{align*}
\]
To check, substitute a value greater than or equal to -3 for \(t\) in the inequality.
\(8 + t \leq 3(t + 4) + 2\)
\[
\begin{align*}
8 + 0 & \leq 3(0 + 4) + 2 \\
8 & \leq 3(4) + 2 \\
8 & \leq 12 + 2 \\
8 & \leq 14 \\
\end{align*}
\]
The solution checks.
7-2 Division Properties of Exponents

79. \(9n + 3(1 - 6n) \leq 21\)

**SOLUTION:**
\[
9n + 3(1 - 6n) \leq 21
\]
\[
9n + 3 - 18n \leq 21
\]
\[
-9n + 3 - 3 \leq 21 - 3
\]
\[
-9n + 3 \leq 21 - 3
\]
\[
\frac{-9n}{-9} \geq \frac{18}{-9}
\]
\[
n \geq -2
\]
To check, substitute a value greater than or equal to \(-2\) for \(n\) in the inequality.
\[
9n + 3[1 - 6n] \leq 21
\]
\[
9(0) + 3[1 - 6(0)] \leq 21
\]
\[
0 + 3(1) \leq 21
\]
\[
3 \leq 21
\]
The solution checks.

80. \(-6(b + 5) > 3(b - 5)\)

**SOLUTION:**
\[
-6(b + 5) > 3(b - 5)
\]
\[
-6b - 30 > 3b - 15
\]
\[
-6b - 3b - 30 > 3b - 3b - 15
\]
\[
-9b - 30 > -15
\]
\[
-9b - 30 + 30 > -15 + 30
\]
\[
-9b < 15
\]
\[
\frac{-9b}{-9} < \frac{15}{-9}
\]
\[
b < -\frac{5}{3}
\]
To check, substitute a value less than \(-\frac{5}{3}\) for \(b\) in the inequality.
\[
-6(b + 5) > 3(b - 5)
\]
\[
-6(-2 + 5) > 3(-2 - 5)
\]
\[
-6(-10) > 3(10)
\]
\[
60 > 30
\]
The solution checks.
7-2 Division Properties of Exponents

81. **GRADES** In a high school science class, a test is worth three times as much as a quiz. What is the student’s average grade?

![Science Grades Table]

**SOLUTION:**
Since the tests are worth three times as much as a quiz, each test is like 3 quizzes. So, in order to find the average grade, we have to count each test 3 times. That makes a total of nine grades. Find the average of the 9 grades.

\[
3(85) + 3(92) + 82 + 75 + 95 = 783 \\
783 \div 9 = 87
\]

The student’s average grade is 87.

**Evaluate each expression.**

82. \(9^2\)

**SOLUTION:**
\[9^2 = 9 \times 9 = 81\]

83. \(11^2\)

**SOLUTION:**
\[11^2 = 11 \times 11 = 121\]

84. \(10^6\)

**SOLUTION:**
\[10^6 = 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000\]

85. \(10^4\)

**SOLUTION:**
\[10^4 = 10 \times 10 \times 10 \times 10 = 10,000\]

86. \(3^5\)

**SOLUTION:**
\[3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243\]
7-2 Division Properties of Exponents

87. \(5^3\)

\[SOLUTION:\]
\[5^3 = 5 \cdot 5 \cdot 5 = 125\]

88. \(12^3\)

\[SOLUTION:\]
\[12^3 = 12 \cdot 12 \cdot 12 = 1728\]

89. \(4^6\)

\[SOLUTION:\]
\[4^6 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 4096\]