5-5 Inequalities Involving Absolute Value

Solve each inequality. Then graph the solution set.

1. \(|a - 5| < 3\)

**SOLUTION:**

**Case 1** \(a - 5\) is positive

\[a - 5 < 3\]
\[a - 5 + 5 < 3 + 5\]
\[a < 8\]

**Case 2** \(a - 5\) is negative

\[-(a - 5) < 3\]
\[a - 5 + 5 > -3 + 5\]
\[a > 2\]

The solution set is \(\{a \mid 2 < a < 8\}\).

2. \(|u + 3| < 7\)

**SOLUTION:**

**Case 1** \(u + 3\) is positive

\[u + 3 < 7\]
\[u + 3 - 3 < 7 - 3\]
\[u < 4\]

**Case 2** \(u + 3\) is negative

\[-(u + 3) < 7\]
\[u + 3 + 3 > -7 - 3\]
\[u > -10\]

The solution set is \(\{u \mid -10 < u < 4\}\).

3. \(|t + 4| \leq -2\)

**SOLUTION:**

\(|t + 4|\) cannot be negative. So it is not possible for \(|t + 4|\) to be less than or equal to \(-2\). Therefore, there is no solution, and the solution set is the empty set, \(\emptyset\). The graph is also empty.

4. \(|c + 2| > -2\)

**SOLUTION:**

Because absolute values are positive, all values of \(|c + 2|\) will be greater than \(-2\).

The solution set is \(\{c \mid c\) is a real number,\}\).
5-5 Inequalities Involving Absolute Value

5. \( |n + 5| \geq 3 \)

**SOLUTION:**

- **Case 1** \( n + 5 \) is positive
  - \( n + 5 \geq 3 \)
  - \( n \geq -2 \)

- **Case 2** \( n + 5 \) is negative
  - \( -(n + 5) \geq 3 \)
  - \( n + 5 \leq -3 \)
  - \( n \leq -8 \)

The solution set is \( \{n|n \leq -8 \text{ or } n \geq -2\} \).

6. \( |p - 2| \geq 8 \)

**SOLUTION:**

- **Case 1** \( p - 2 \) is positive
  - \( p - 2 \geq 8 \)
  - \( p \geq 10 \)

- **Case 2** \( p - 2 \) is negative
  - \( -(p - 2) \geq 8 \)
  - \( p - 2 \leq -8 \)
  - \( p \leq -6 \)

The solution set is \( \{p|p \leq -6 \text{ or } p \geq 10\} \).
5-5 Inequalities Involving Absolute Value

7. **FINANCIAL LITERACY**  Jerome bought stock in a restaurant chain at $70.85. The stock price ranged from within $0.75 of his purchase price that day. Find the range of prices for which the stock could trade that day.

**SOLUTION:**

\[ |m - 70.85| \leq 0.75 \]

**Case 1** \( m - 70.85 \) is positive

\[
\begin{align*}
m - 70.85 & \leq 0.75 \\
m - 70.85 + 70.85 & \leq 0.75 + 70.85 \\
m & \leq 71.60
\end{align*}
\]

and

**Case 2** \( m - 70.85 \) is negative

\[
\begin{align*}
-(m - 70.85) & \leq 0.75 \\
-m + 70.85 & \leq 0.75 \\
-m + 70.85 - 70.85 & \leq 0.75 - 70.85 \\
-m & \geq -70.10 \\
-m & \geq -70.10
\end{align*}
\]

\[ m \geq 70.10 \]

The solution set is \( \{m \mid 70.10 \leq m \leq 71.60\} \).

8. \(|x + 8| < 16\)

**SOLUTION:**

**Case 1** \( x + 8 \) is positive

\[
\begin{align*}
x + 8 & < 16 \\
x + 8 - 8 & < 16 - 8 \\
x & < 8
\end{align*}
\]

**Case 2** \( x + 8 \) is negative

\[
\begin{align*}
-(x + 8) & < 16 \\
-x - 8 & < 16 \\
-x - 8 + 8 & < 16 + 8 \\
-x & > 24 \\
-1 & > -1 \\
x & > -24
\end{align*}
\]

The solution set is \( \{x \mid -24 < x < 8\} \).
5-5 Inequalities Involving Absolute Value

9. \(|r + 1| \leq 2\)

**SOLUTION:**

**Case 1** \(r + 1\) is positive

\[
\begin{align*}
    r + 1 & \leq 2 \\
    r + 1 - 1 & \leq 2 - 1 \\
    r & \leq 1
\end{align*}
\]

**Case 2** \(r + 1\) is negative

\[
\begin{align*}
    -(r + 1) & \leq 2 \\
    -r - 1 & \leq 2 \\
    -r - 1 + 1 & \leq 2 + 1 \\
    -r & \geq 3 \\
    -\frac{r}{-1} & \geq \frac{3}{-1} \\
    r & \geq -3
\end{align*}
\]

The solution set is \(\{r | -3 \leq r \leq 1\}\).

10. \(|2c - 1| \leq 7\)

**SOLUTION:**

**Case 1** \(2c - 1\) is positive

\[
\begin{align*}
    2c - 1 & \leq 7 \\
    2c - 1 + 1 & \leq 7 + 1 \\
    2c & \leq 8 \\
    2c & \leq \frac{8}{2} \\
    c & \leq 4
\end{align*}
\]

and

**Case 2** \(2c - 1\) is negative

\[
\begin{align*}
    -(2c - 1) & \leq 7 \\
    -2c + 1 & \leq 7 \\
    -2c + 1 - 1 & \leq 7 - 1 \\
    -2c & \leq 6 \\
    \frac{-2c}{-2} & \geq \frac{6}{-2} \\
    c & \geq -3
\end{align*}
\]

The solution set is \(\{c | -3 \leq c \leq 4\}\).
5-5 Inequalities Involving Absolute Value

11. $|3h - 3| < 12$

**SOLUTION:**

**Case 1** $3h - 3$ is positive

$3h - 3 < 12$
$3h - 3 + 3 < 12 + 3$
$3h < 15$
$\frac{3h}{3} < \frac{15}{3}$
$h < 5$

and

**Case 2** $3h - 3$ is negative

$-(3h - 3) < 12$
$-3h + 3 < 12$
$-3h + 3 - 3 < 12 - 3$
$-3h < 9$
$\frac{-3h}{-3} > \frac{9}{-3}$
$h > -3$

The solution set is $\{h | -3 < h < 5\}$.

12. $|m + 4| < -2$

**SOLUTION:**

$|m + 4|$ cannot be negative. So it is not possible for $|m + 4|$ to be less than $-2$. Therefore, there is no solution, and the solution set is the empty set, $\emptyset$. The graph is also empty.

13. $|w + 5| < -8$

**SOLUTION:**

$|w + 5|$ cannot be negative. So it is not possible for $|w + 5|$ to be less than $-8$. Therefore, there is no solution, and the solution set is the empty set, $\emptyset$. The graph is also empty.
5-5 Inequalities Involving Absolute Value

14. \( |r + 2| > 6 \)

**SOLUTION:**

Case 1 \( r + 2 \) is positive

\[
\begin{align*}
  r + 2 &> 6 \\
  r + 2 - 2 &> 6 - 2 \\
  r &> 4 
\end{align*}
\]

Case 2 \( r + 2 \) is negative

\[
\begin{align*}
  -(r + 2) &> 6 \\
  -r + 2 &< -6 \\
  -r + 2 - 2 &< -6 - 2 \\
  -r &< -8 \\
  r &> -8 
\end{align*}
\]

The solution set is \( \{ r \mid r < -8 \text{ or } r > 4 \} \).

15. \( |k - 4| > 3 \)

**SOLUTION:**

Case 1 \( k - 4 \) is positive

\[
\begin{align*}
  k - 4 &> 3 \\
  k - 4 + 4 &> 3 + 4 \\
  k &> 7 
\end{align*}
\]

Case 2 \( k - 4 \) is negative

\[
\begin{align*}
  -(k - 4) &> 3 \\
  -k + 4 &< -3 \\
  -k + 4 + 4 &< -3 + 4 \\
  -k &< 1 \\
  k &> 1 
\end{align*}
\]

The solution set is \( \{ k \mid k < 1 \text{ or } k > 7 \} \).

16. \( |2h - 3| \geq 9 \)

**SOLUTION:**

Case 1 \( 2h - 3 \) is positive

\[
\begin{align*}
  2h - 3 &\geq 9 \\
  2h - 3 + 3 &\geq 9 + 3 \\
  2h &\geq 12 \\
  \frac{2h}{2} &\geq \frac{12}{2} \\
  h &\geq 6 
\end{align*}
\]

Case 2 \( 2h - 3 \) is negative

\[
\begin{align*}
  -(2h - 3) &\geq 9 \\
  -2h + 3 &\geq 9 \\
  -2h + 3 - 3 &\geq 9 - 3 \\
  -2h &\geq 6 \\
  \frac{-2h}{-2} &\leq \frac{6}{-2} \\
  h &\leq -3 
\end{align*}
\]

The solution set is \( \{ h \mid h \leq -3 \text{ or } h \geq 6 \} \).
5-5 Inequalities Involving Absolute Value

17. \(|4p + 2| \geq 10\)

**SOLUTION:**

**Case 1** \(4p + 2\) is positive

\[
4p + 2 \geq 10 \\
4p + 2 - 2 \geq 10 - 2 \\
4p \geq 8 \\
\frac{4p}{4} \geq \frac{8}{4} \\
p \geq 2
\]

or

**Case 2** \(4p + 2\) is negative

\[
-(4p + 2) \geq 10 \\
-4p - 2 \geq 10 \\
-4p - 2 + 2 \geq 10 + 2 \\
-4p \geq 12 \\
\frac{-4p}{-4} \leq \frac{12}{-4} \\
p \leq -3
\]

The solution set is \(\{p \mid p \leq -3 \text{ or } p \geq 2\}\).

18. \(|5v + 3| > -9\)

**SOLUTION:**

Because absolute values are positive, all values of \(|5v + 3|\) will be greater than \(-9\). The solution set is \(\{v \mid v \text{ is a real number}\}\).

19. \(|-2c - 3| > -4\)

**SOLUTION:**

Because absolute values are positive, all values of \(|-2c - 3|\) will be greater than \(-4\). The solution set is \(\{c \mid c \text{ is a real number}\}\).
5-5 Inequalities Involving Absolute Value

20. SCUBA DIVING  The pressure of a scuba tank should be within 500 pounds per square inch (psi) of 2500 psi. Write the range of optimum pressures.

SOLUTION:
The range of optimum pressures for scuba tanks is between 2500 psi plus 500 psi and 2500 psi minus 500 psi. The optimum range can be expressed as \{p \mid 2000 \leq p \leq 3000\}.

Solve each inequality. Then graph the solution set.

21. \(|4n + 3| \geq 18\)

SOLUTION:

Case 1 4n + 3 is positive

\[4n + 3 \geq 18\]
\[4n \geq 15\]
\[n \geq \frac{15}{4}\]

Case 2 4n + 3 is negative

\[-(4n + 3) \geq 18\]
\[-4n - 3 \geq 18\]
\[-4n \geq 21\]
\[n \leq -\frac{21}{4}\]

The solution set is \(\{n \mid n \leq -\frac{21}{4} \text{ or } n \geq \frac{15}{4}\}\).

22. \(|5t - 2| \leq 6\)

SOLUTION:

Case 1 5t - 2 is positive

\[5t - 2 \leq 6\]
\[5t \leq 8\]
\[t \leq \frac{8}{5}\]

Case 2 5t - 2 is negative

\[-(5t - 2) \leq 6\]
\[-5t + 2 \leq 6\]
\[t \geq \frac{4}{5}\]

The solution set is \(\{t \mid -\frac{4}{5} \leq t \leq \frac{8}{5}\}\).
5-5 Inequalities Involving Absolute Value

23. \[ \left| \frac{3h+1}{2} \right| < 8 \]

**SOLUTION:**

**Case 1** \( \frac{3h+1}{2} \) is positive

\[ \frac{3h+1}{2} < 8 \]
\[ 2 \left( \frac{3h+1}{2} \right) < 2(8) \]
\[ 3h + 1 < 16 \]
\[ 3h + 1 - 1 < 16 - 1 \]
\[ 3h < 15 \]
\[ \frac{3h}{3} < \frac{15}{3} \]
\[ h < 5 \]

and

**Case 2** \( \frac{3h+1}{2} \) is negative

\[ -\left( \frac{3h+1}{2} \right) < 8 \]
\[ \frac{3h+1}{2} > -8 \]
\[ 2 \left( \frac{3h+1}{2} \right) > 2(-8) \]
\[ 3h + 1 > -16 \]
\[ 3h + 1 - 1 > -16 - 1 \]
\[ 3h > -17 \]
\[ \frac{3h}{3} > \frac{-17}{3} \]
\[ h > -\frac{17}{3} \]

The solution set is \( \left\{ h \left| -\frac{17}{3} < h < 5 \right. \right\} \).
24. \( \left| \frac{2p-8}{4} \right| \geq 9 \)

**SOLUTION:**

**Case 1** \( \frac{2p-8}{4} \) is positive

\[
\frac{2p-8}{4} \geq 9
\]

\[
4 \left( \frac{2p-8}{4} \right) \geq 4 \cdot 9
\]

\[
2p-8 \geq 36
\]

\[
2p \geq 44
\]

\[
\frac{2p}{2} \geq \frac{44}{2}
\]

\[
p \geq 22
\]

or

**Case 2** \( \frac{2p-8}{4} \) is negative

\[
- \left( \frac{2p-8}{4} \right) \geq 9
\]

\[
- \frac{2p-8}{4} \leq -9
\]

\[
4 \left( -\frac{2p-8}{4} \right) \leq 4 \cdot (-9)
\]

\[
2p-8 \leq -36
\]

\[
2p \leq -28
\]

\[
\frac{2p}{2} \leq \frac{-28}{2}
\]

\[
p \leq -14
\]

The solution set is \( \{ p \mid p \leq -14 \text{ or } p \geq 22 \} \).
5-5 Inequalities Involving Absolute Value

25. \[ \left| \frac{7c + 3}{2} \right| \leq -5 \]

**SOLUTION:**

\[ \left| \frac{7c + 3}{2} \right| \] cannot be negative. So it is not possible for \( \left| \frac{7c + 3}{2} \right| \) to be less than or equal to \(-5\). Therefore, there is no solution, and the solution set is the empty set, \( \emptyset \). The graph is also empty.

\[ \begin{array}{ccccccc}
-5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\end{array} \]

26. \[ \left| \frac{2g + 3}{2} \right| > -7 \]

**SOLUTION:**

Because absolute values are positive, all values of \( \left| \frac{2g + 3}{2} \right| \) will be greater than \(-7\).

The solution set is \{ \( g \) | \( g \) is a real number. \}.

\[ \begin{array}{ccccccc}
-5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\end{array} \]
27. \(|-6r - 4| < 8\)

**SOLUTION:**

**Case 1** \(-6r - 4\) is positive

\[-6r - 4 < 8\]
\[-6r - 4 + 4 < 8 + 4\]
\[-6r < 12\]
\[\frac{-6r}{-6} > \frac{12}{-6}\]
\[r > -2\]

and

**Case 2** \(-6r - 4\) is negative

\[-(-6r - 4) < 8\]
\[6r + 4 < 8\]
\[6r + 4 - 4 < 8 - 4\]
\[6r < 4\]
\[\frac{6r}{6} < \frac{4}{6}\]
\[r < \frac{2}{3}\]

The solution set is \(\left\{r \mid -2 < r < \frac{2}{3}\right\}\).
28. \(|-3p - 7| > 5\)

**SOLUTION:**

**Case 1** \(-3p - 7\) is positive

\[-3p - 7 > 5\]
\[-3p - 7 + 7 > 5 + 7\]
\[-3p > 12\]
\[-\frac{3p}{-3} < \frac{12}{-3}\]
\[p < -4\]

or

**Case 2** \(-3p - 7\) is negative

\[-( -3p - 7) > 5\]
\[3p + 7 > 5\]
\[3p + 7 - 7 > 5 - 7\]
\[3p > -2\]
\[\frac{3p}{3} > \frac{-2}{3}\]
\[p > -\frac{2}{3}\]

The solution set is \(\{p \mid p < -4 \text{ or } p > -\frac{2}{3}\}\).  

![Graph showing the solution set for the inequality |−3p − 7| > 5](image.png)
5-5 Inequalities Involving Absolute Value

29. \(|-h + 1.5| < 3\)

**SOLUTION:**

**Case 1** \(-h + 1.5\) is positive

\[-h + 1.5 < 3\]

\[-h + 1.5 - 1.5 < 3 - 1.5\]

\[-h < 1.5\]

\[h > -1.5\]

and

**Case 2** \(-h + 1.5\) is negative

\[-(-h + 1.5) < 3\]

\[h - 1.5 < 3\]

\[h - 1.5 + 1.5 < 3 + 1.5\]

\[h < 4.5\]

The solution set is \(\{h \mid -1.5 < h < 4.5\}\).

30. **MUSIC DOWNLOADS** Kareem is allowed to download $10 worth of music each month. This month he has spent within $3 of his allowance.

a. What is the range of money he has spent on music downloads this month?

b. Graph the range of the money that he spent.

**SOLUTION:**

a. The range of money he has spent on music downloads is between $10 plus $3 and $10 minus $3. The range can be expressed as \(\{m \mid 7 \leq m \leq 13\}\).

b.

31. **CHEMISTRY** Water can be present in our atmosphere as a solid, liquid, or gas. Water freezes at 32°F and vaporizes at 212°F.

a. Write the range of temperatures in which water is not a liquid.

b. Graph this range.

c. Write the absolute value inequality that describes this situation.

**SOLUTION:**

a. The range of temperatures that water is not a liquid include 90° below 122° and 90° above 122°. The range can be expressed as \(\{t \mid t < 32 \text{ or } t > 212\}\).

b.

c. The difference between the temperature of water that is not a liquid and 122° should be more than 90°. The absolute value inequality that describes this situation is \(|t - 122| > 90\).
**5-5 Inequalities Involving Absolute Value**

**Write an open sentence involving absolute value for each graph.**

32. [Graph: Three lines pointing to the right, starting at -5, -3, and 0, with shading to the right of each line.]

**SOLUTION:**
Since the endpoints are circles at -2 and 2 and the shaded area is between the circles, the sign for this sentence is less than.

The midpoint between -2 and 2 is 0. The distance between the midpoint and each endpoint is 2. So, the equation is 
\[ |x - 0| < 2 \text{ or } |x| < 2. \]

33. [Graph: Three horizontal lines, with shading to the left of the middle line and above the top line.]

**SOLUTION:**
Since the endpoints are dots at -5 and 3 and the shaded area is between the dots, the sign for this sentence is less than or equal to.

The midpoint between -5 and 3 is -1. The distance between the midpoint and each endpoint is 4. So, the equation is 
\[ |x - (-1)| \leq 4 \text{ or } |x + 1| \leq 4. \]

34. [Graph: Three horizontal lines, with shading to the left of the middle line and below the bottom line.]

**SOLUTION:**
Since the endpoints are dots at -3 and 1 and the shaded areas are outside of the dots, the sign for this sentence is greater than or equal to.

The midpoint between -3 and 1 is -1. The distance between the midpoint and each endpoint is 2. So, the equation is 
\[ |x - (-1)| \geq 2 \text{ or } |x + 1| \geq 2. \]

35. [Graph: A line from 0 to 11, with shading to the right.]

**SOLUTION:**
Since the endpoints are circles at 1 and 10 and the shaded areas are outside of the circles, the sign for this sentence is greater than.

The midpoint between 1 and 10 is 5.5. The distance between the midpoint and each endpoint is 4.5. So, the equation is 
\[ |x - (5.5)| > 4.5 \text{ or } |x - 5.5| > 4.5. \]

36. **ANIMALS** A sheep's normal body temperature is 39°C. However, a healthy sheep may have body temperatures 1°C above or below this temperature. What is the range of body temperatures for a sheep?

**SOLUTION:**
The range of normal sheep body temperature is between 39°C plus 1°C and 39°C minus 1°C. The range can be expressed as \{t | 38 ≤ t ≤ 40\}. 
5-5 Inequalities Involving Absolute Value

37. MINIATURE GOLF  Ginger’s score was within 5 strokes of her average score of 52. Determine the range of scores for Ginger’s game.

SOLUTION:
The range of scores for Ginger’s game is between 52 plus 5 strokes and 52 minus 5 strokes. The range can be expressed as \{g | 47 \leq g \leq 57\}.

Express each statement using an inequality involving absolute value. Do not solve.

38. The pH of a swimming pool must be within 0.3 of a pH of 7.5.

SOLUTION:
Since within implies that the pH can be up to 0.3 of a pH away from the midpoint of 7.5 in either direction, the symbol to use is less than or equal to.

In other words, the distance between the pH level \( p \), and the midpoint 7.5, is less than or equal to 0.3.

So the inequality is \( |p - 7.5| \leq 0.3 \).

39. The temperature inside a refrigerator should be within 1.5 degrees of 38°F.

SOLUTION:
Since within implies that the temperature can be up to 1.5 degrees away from the midpoint of 38 degrees in either direction, the symbol to use is less than or equal to.

In other words, the distance between the temperature \( t \), and the midpoint 38, is less than or equal to 1.5.

So the inequality is \( |t - 38| \leq 1.5 \).

40. Ramona’s bowling score was within 6 points of her average score of 98.

SOLUTION:
Since within implies that Ramona’s score can be up to 6 points away from her average of 98 points in either direction, the symbol to use is less than or equal to.

In other words, the distance between the bowling score \( b \), and her average score 98, is less than or equal to 6.

So the inequality is \( |b - 98| \leq 6 \).

41. The cruise control of a car should keep the speed within 3 miles per hour of 55.

SOLUTION:
Since within implies that the cruise control speed can be up to 3 miles per hour away from the average of 55 miles per hour in either direction, the symbol to use is less than or equal to.

In other words, the distance between the cruise control speed \( c \), and the average 55, is less than or equal to 3.

So the inequality is \( |c - 55| \leq 3 \).
5-5 Inequalities Involving Absolute Value

42. MULTIPLE REPRESENTATIONS In this problem, you will investigate the graphs of absolute value inequalities on a coordinate plane.

a. TABULAR Copy and complete the table. Substitute the x and \( f(x) \) values for each point into each inequality. Mark whether the resulting statement is true or false.

| Point | \( f(x) \geq |x - 1| \) | true/false | \( f(x) \leq |x - 1| \) | true/false |
|-------|--------------------------|------------|--------------------------|------------|
| (-4, 2) | 2 \geq 5 | false | 2 \leq 5 | true |
| (-2, 2) | 2 \geq 3 | false | 2 \leq 3 | true |
| (0, 2) | 2 \geq 1 | true | 2 \leq 1 | false |
| (2, 2) | 2 \geq 1 | true | 2 \leq 1 | false |
| (4, 2) | 2 \geq 3 | false | 2 \leq 3 | true |

b. GRAPHICAL Graph \( f(x) = |x - 1| \).

c. GRAPHICAL Plot each point from the table that made \( f(x) \geq |x - 1| \) a true statement, on the graph in red. Plot each point that made \( f(x) \leq |x - 1| \) a true statement, in blue.

d. LOGICAL Make a conjecture about what the graphs about \( f(x) \geq |x - 1| \) and \( f(x) \leq |x - 1| \) look like. Complete the table with other points to verify your conjecture.

e. GRAPHICAL Use what you discovered to graph \( f(x) \geq |x - 3| \).

SOLUTION:

| Point | \( f(x) \geq |x - 1| \) | true/false | \( f(x) \leq |x - 1| \) | true/false |
|-------|--------------------------|------------|--------------------------|------------|
| (-4, 2) | 2 \geq 5 | false | 2 \leq 5 | true |
| (-2, 2) | 2 \geq 3 | false | 2 \leq 3 | true |
| (0, 2) | 2 \geq 1 | true | 2 \leq 1 | false |
| (2, 2) | 2 \geq 1 | true | 2 \leq 1 | false |
| (4, 2) | 2 \geq 3 | false | 2 \leq 3 | true |

b. Since \( f(x) \) cannot be negative, the minimum point of the graph is where \( f(x) = 0 \).

\[
f(x) = |x - 1|
\]

\[
0 = x - 1
\]

\[
1 = x
\]

Make a table of values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
The slope is 1 and the left 1 unit. Plot the point. Draw a line through the two points.

The slope is 2 and the right 1 unit. Plot the point. Draw a line through the two points.

The slope is 4 and the

Check.

First set up the inequality. The service fee plus the product of the number of withdrawals and the amount of each

So, the correct choice is B.

So the inequality is

Since

In other words, the distance between the temperature

Since the endpoints are dots at

Since the endpoints are circles at

b.

F

The midpoint between

The solution set is

or

F

Case 2

The solution set is


e. \( f(x) \geq |x - 3| \)

Since \( f(x) \) cannot be negative, the minimum point of the graph is where \( f(x) = 0 \).

\[
f(x) = |x - 3|
\]

\[
0 = x - 3
\]

\[
3 = x
\]
5-5 Inequalities Involving Absolute Value

So the minimum point of the graph would occur at \( x = 3 \).

Using a table of values we can graph more of \( f(x) = |x - 3| \). Since only points above the graph make the statement true, the region above the graph would be shaded. Because the inequality symbol is \( \geq \), the boundary is included in the solution. This is shown as a solid line.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

43. **ERROR ANALYSIS** Lucita sketched a graph of her solution to \( |2a - 3| > 1 \). Is she correct? Explain your reasoning.

**SOLUTION:**
Sample answer: Lucita’s graph is not correct. She forgot to change the direction of the inequality sign for the negative case of the absolute value. The arrow from 1 should point to the left.

44. **REASONING** The graph of an absolute value inequality is sometimes, always, or never the union of two graphs. Explain.

**SOLUTION:**
Sometimes; the graph could be the intersection of two graphs, the empty set, or all real numbers.

45. **CHALLENGE** Demonstrate why the solution of \( |t| > 0 \) is not all real numbers. Explain your reasoning.

**SOLUTION:**
Sample answer: If \( t = 0 \), then the absolute value is equal to 0, not greater than 0.
5-5 Inequalities Involving Absolute Value

46. **OPEN ENDED** Write an absolute value inequality to represent a real-world situation. Interpret the solution.

**SOLUTION:**
Sample answer: The range of normal body temperature of a healthy human varies plus or minus 1.4° from 98.6°. Write the inequality to show the normal range of human body temperature. 
\[ |t - 98.6| < 1.4 \]

**Case 1** \( t - 98.6 \) is positive

\[
t - 98.6 < 1.4
\]
\[
t - 98.6 + 98.6 < 1.4 + 98.6
\]
\[
t < 100
\]
and

**Case 2** \( t - 98.6 \) is negative

\[
-(t - 98.6) < 1.4
\]
\[
-t + 98.6 < 1.4
\]
\[
-t + 98.6 - 98.6 < 1.4 - 98.6
\]
\[
-t < -97.2
\]
\[
\frac{-t}{-1} > \frac{-97.2}{-1}
\]
\[
t > 97.2
\]

The solution set is \( \{t|97.2 < t < 100\} \).
The normal body temperature of a healthy human is between 97.2° and 100°.

47. **WRITING IN MATH** Explain how to determine whether an absolute value inequality uses a compound inequality with **and** or a compound inequality with **or**. Then summarize how to solve absolute value inequalities.

**SOLUTION:**
Sample answer: When an absolute value is on the left and the inequality symbol is \( < \) or \( \leq \), the compound sentence uses **and**, and if the inequality symbol is \( > \) or \( \geq \), the compound sentence uses **or**. To solve, if \( |x| < n \), then set up and solve the inequalities \( x < n \) and \( x > -n \), and if \( |x| > n \), then set up and solve the inequalities \( x > n \) or \( x < -n \).
48. The formula for acceleration in a circle is \( a = \frac{v^2}{r} \). Which of the following shows the equation solved for \( r \)?

A  \( r = v \)
B  \( r = \frac{v^2}{a} \)
C  \( r = av^2 \)
D  \( r = \frac{\sqrt{a}}{v} \)

**SOLUTION:**
\[
\begin{align*}
a &= \frac{v^2}{r} \\
ar &= r \left( \frac{v^2}{r} \right) \\
ar &= v^2 \\
a &= \frac{v^2}{a} \\
r &= \frac{v^2}{a}
\end{align*}
\]
So, the correct choice is B.
5-5 Inequalities Involving Absolute Value

49. An engraver charges a $3 set-up fee and $0.25 per word. Which table shows the total price $p$ for $w$ words?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F</strong></td>
<td></td>
</tr>
<tr>
<td>$w$</td>
<td>$p$</td>
</tr>
<tr>
<td>15</td>
<td>$3$</td>
</tr>
<tr>
<td>20</td>
<td>$4.25$</td>
</tr>
<tr>
<td>25</td>
<td>$5.50$</td>
</tr>
<tr>
<td>30</td>
<td>$7.75$</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>G</strong></td>
<td></td>
</tr>
<tr>
<td>$w$</td>
<td>$p$</td>
</tr>
<tr>
<td>15</td>
<td>$6.75$</td>
</tr>
<tr>
<td>20</td>
<td>$7$</td>
</tr>
<tr>
<td>25</td>
<td>$7.25$</td>
</tr>
<tr>
<td>30</td>
<td>$7.50$</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>H</strong></td>
<td></td>
</tr>
<tr>
<td>$w$</td>
<td>$p$</td>
</tr>
<tr>
<td>15</td>
<td>$3.75$</td>
</tr>
<tr>
<td>20</td>
<td>$5$</td>
</tr>
<tr>
<td>25</td>
<td>$6.25$</td>
</tr>
<tr>
<td>30</td>
<td>$8.50$</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>J</strong></td>
<td></td>
</tr>
<tr>
<td>$w$</td>
<td>$p$</td>
</tr>
<tr>
<td>15</td>
<td>$6.75$</td>
</tr>
<tr>
<td>20</td>
<td>$8$</td>
</tr>
<tr>
<td>25</td>
<td>$9.25$</td>
</tr>
<tr>
<td>30</td>
<td>$10.50$</td>
</tr>
</tbody>
</table>

**SOLUTION:**

$p = 3 + 0.25w$

Substitute the $w$ values into the equation.

$p = 3 + 0.25w$

$= 3 + 0.25(15)$

$= 3 + 6.75$

$= 6.75$

Choices F and H have the wrong value of $p$ for $w = 15$.

$p = 3 + 0.25w$

$= 3 + 0.25(20)$

$= 3 + 8$

$= 8$

Choice G has the wrong value of $p$ for $w = 20$.

So, the correct choice is J.
5-5 Inequalities Involving Absolute Value

50. **SHORT RESPONSE** The table shows the items in stock at the school store the first day of class. What is the probability that an item chosen at random was a notebook?

<table>
<thead>
<tr>
<th>Item</th>
<th>Number Purchased</th>
</tr>
</thead>
<tbody>
<tr>
<td>pencil</td>
<td>57</td>
</tr>
<tr>
<td>pen</td>
<td>38</td>
</tr>
<tr>
<td>eraser</td>
<td>6</td>
</tr>
<tr>
<td>folder</td>
<td>25</td>
</tr>
<tr>
<td>notebook</td>
<td>18</td>
</tr>
</tbody>
</table>

**SOLUTION:**
Find the total number of purchases.

\[57 + 38 + 6 + 25 + 18 = 144\]

The probability of a notebook being picked is \[\frac{18}{144} = \frac{1}{8}\].

51. Solve for \(n\). \(|2n - 3| = 5\)
   A \{-4, -1\}
   B \{-1, 4\}
   C \{1, 1\}
   D \{4, 4\}

**SOLUTION:**

**Case 1** \(2n - 3\) is positive

\[2n - 3 = 5\]
\[2n = 8\]
\[n = 4\]

**Case 2** \(2n - 3\) is negative

\[-(2n - 3) = 5\]
\[-2n + 3 = 5\]
\[-2n = 2\]
\[n = -1\]

The solution set is \{-1, 4\}. So the correct choice is B.

**Solve each compound inequality. Then graph the solution set.**

52. \(b + 3 < 11\) and \(b + 2 > -3\)

**SOLUTION:**

\[b + 3 < 11\]  and  \[b + 2 > -3\]
\[b + 3 - 3 < 11 - 3\]
\[b + 2 - 2 > -3 - 2\]
\[b < 8\]
\[b > -5\]

The solution set is \(\{b\} | -5 < b < 8\}.

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5-5 Inequalities Involving Absolute Value

53. \(6 \leq 2t - 4 \leq 8\)

**SOLUTION:**

First, express \(6 \leq 2t - 4 \leq 8\) using *and*. Then solve each inequality.

\[
\begin{align*}
6 \leq 2t - 4 & \quad \text{and} \quad 2t - 4 \leq 8 \\
6 + 4 \leq 2t - 4 + 4 & \quad 2t - 4 + 4 \leq 8 + 4 \\
10 \leq 2t & \quad 2t \leq 12 \\
10 \leq 2t & \quad 2t \leq 12 \\
5 \leq t & \quad t \leq 6
\end{align*}
\]

The solution set is \(\{t | 5 \leq t \leq 6\}\).

54. \(2c - 3 \geq 5\) or \(3c + 7 \leq -5\)

**SOLUTION:**

\[
\begin{align*}
2c - 3 \geq 5 & \quad \text{or} \quad 3c + 7 \leq -5 \\
2c - 3 + 3 \geq 5 + 3 & \quad 3c + 7 - 7 \leq -5 - 7 \\
2c \geq 8 & \quad 3c \leq -12 \\
2c \geq 8 & \quad 3c \leq -12 \\
\frac{2c}{2} \geq \frac{8}{2} & \quad \frac{3c}{3} \leq \frac{-12}{3} \\
c \geq 4 & \quad c \leq -4
\end{align*}
\]

The solution set is \(\{c | c \geq 4 \text{ or } c \leq -4\}\).
5-5 Inequalities Involving Absolute Value

55. **GEOMETRY**  One angle of a triangle measures 10° more than the second. The measure of the third angle is twice the sum of the measure of the first two angles. Find the measure of each angle.

**SOLUTION:**
Let \( a \) = the measurement of the second angle.
1st angle:  \( a + 10 \)
3rd angle:  \( 2(\text{2nd angle} + \text{1st angle}) = 2[a + (a + 10)] \)

\[
\begin{align*}
\frac{a + (a + 10) + 2[a + (a + 10)]}{a + a + 10 + 2(2a + 10)} &= 180 \\
\frac{2a + 10 + 4a + 20}{6a + 30} &= 180 \\
6a + 30 &= 180 \\
6a &= 150 \\
a &= 25
\end{align*}
\]

The measurement of the second angle is 25, so solve for the other two angles.

1st angle = 25 + 10 = 35
3rd angle = 2(25 + 35) = 120

So the three angles are 25°, 35°, and 120°.

56. **FINANCIAL LITERACY**  Jackson’s bank charges him a monthly service fee of $6 for his checking account and $2 for each out-of-network ATM withdrawal. Jackson’s account balance is $87. Write and solve an inequality to find how many out-of-network ATM withdrawals of $20 Jackson can make without overdrawing his account.

**SOLUTION:**
First set up the inequality. The service fee plus the product of the number of withdrawals and the amount of each withdrawal is less than or equal to the amount in the account.

\[
6 + w(20 + 2) \leq 87 \quad \text{Substitute}.
\]

\[
6 + 22w \leq 87 \quad \text{Simplify}.
\]

\[
22w \leq 81 \quad \text{Subtract}.
\]

\[
w \leq \frac{81}{22} \quad \text{Divide}.
\]

\[
w \leq 3.636 \quad \text{Simplify}.
\]

Jackson can make up to 3 withdrawals.
5-5 Inequalities Involving Absolute Value

Solve each equation. Then check your solution.

57. \( c - 7 = 11 \)

**SOLUTION:**

Solve.
\[
\begin{align*}
  c - 7 &= 11 \\
  c - 7 + 7 &= 11 + 7 \\
  c &= 18
\end{align*}
\]

Check.
\[
\begin{align*}
  c - 7 &= 11 \\
  18 - 7 &= 11 \\
  11 &= 11
\end{align*}
\]

58. \( 2w = 24 \)

**SOLUTION:**

Solve.
\[
\begin{align*}
  2w &= 24 \\
  \frac{2w}{2} &= \frac{24}{2} \\
  w &= 12
\end{align*}
\]

Check.
\[
\begin{align*}
  2w &= 24 \\
  2(12) &= 24 \\
  24 &= 24
\end{align*}
\]

59. \( 9 + p = -11 \)

**SOLUTION:**

Solve.
\[
\begin{align*}
  9 + p &= -11 \\
  9 - 9 + p &= -11 - 9 \\
  p &= -20
\end{align*}
\]

Check.
\[
\begin{align*}
  9 + p &= -11 \\
  9 + (-20) &= -11 \\
  -11 &= -11
\end{align*}
\]
5-5 Inequalities Involving Absolute Value

60. \( \frac{t}{5} = 20 \)

**SOLUTION:**
Solve.

\[ \frac{t}{5} = 20 \]

\[ 5 \left( \frac{t}{5} \right) = 5(20) \]

\[ t = 100 \]

Check.

\[ \frac{t}{5} = 20 \]

\[ 100 \]

\[ \frac{5}{5} = 20 \]

\[ 20 = 20 \]

**Graph each equation.**

61. \( y = 4x - 1 \)

**SOLUTION:**
The slope is 4 and the y-intercept is \(-1\). To graph the equation, plot the y-intercept (0, \(-1\)). Then move up 4 units and right 1 unit. Plot the point. Draw a line through the two points.

![Graph of y = 4x - 1](image)
5-5 Inequalities Involving Absolute Value

62. \( y - x = 3 \)

**SOLUTION:**
Write the equation in slope-intercept form.

\[
\begin{align*}
y - x &= 3 \\
y - x + x &= 3 + x \\
y &= x + 3
\end{align*}
\]

The slope is 1 and the \( y \)-intercept is 3. To graph the equation, plot the \( y \)-intercept (0, 3). Then move up 1 unit and right 1 unit. Plot the point. Draw a line through the two points.

![Graph of \( y - x = 3 \)]

63. \( 2x - y = -4 \)

**SOLUTION:**
Write the equation in slope-intercept form.

\[
\begin{align*}
2x - y &= -4 \\
2x - 2x - y &= -4 - 2x \\
- y &= -4 - 2x \\
\frac{- y}{-1} &= \frac{-2x - 4}{-1} \\
y &= 2x + 4
\end{align*}
\]

The slope is 2 and the \( y \)-intercept is 4. To graph the equation, plot the \( y \)-intercept (0, 4). Then move down 2 units and left 1 unit. Plot the point. Draw a line through the two points.

![Graph of \( 2x - y = -4 \)]
5-5 Inequalities Involving Absolute Value

64. $3y + 2x = 6$

**SOLUTION:**
Write the equation in slope-intercept form.

\[
\begin{align*}
3y + 2x &= 6 \\
3y &= -2x + 6 \\
y &= -\frac{2}{3}x + 2
\end{align*}
\]

The slope is $-\frac{2}{3}$ and the y-intercept is 2. To graph the equation, plot the y-intercept $(0, 2)$. Then move down 2 units and right 3 units. Plot the point. Draw a line through the two points.

![Graph of 3y + 2x = 6](image)

65. $4y = 4x - 16$

**SOLUTION:**
Write the equation in slope-intercept form.

\[
\begin{align*}
4y &= 4x - 16 \\
4y &= 4x - 16 \\
y &= x - 4
\end{align*}
\]

The slope is 1 and the y-intercept is $-4$. To graph the equation, plot the y-intercept $(0, -4)$. Then move up 1 unit and right 1 unit. Plot the point. Draw a line through the two points.

![Graph of 4y = 4x - 16](image)
5-5 Inequalities Involving Absolute Value

66. \(2y - 2x = 8\)

**SOLUTION:**
Write the equation in slope-intercept form.

\[
\begin{align*}
2y - 2x &= 8 \\
2y - 2x + 2x &= 8 + 2x \\
2y &= 2x + 8 \\
\frac{2y}{2} &= \frac{2x + 8}{2} \\
y &= x + 4
\end{align*}
\]

The slope is 1 and the y-intercept is 4. To graph the equation, plot the y-intercept (0, 4). Then move up 1 unit and right 1 unit. Plot the point. Draw a line through the two points.
5-5 Inequalities Involving Absolute Value

67. \(-9 = -3x - y\)

**SOLUTION:**
Write the equation in slope-intercept form.

\[-9 = -3x - y\]
\[-9 + y = -3x - y + y\]
\[-9 + y = -3x\]
\[-9 + y + 9 = -3x + 9\]
\[y = -3x + 9\]

The slope is \(-3\) and the \(y\)-intercept is 9. To graph the equation, plot the \(y\)-intercept (0, 9). Then move down 3 units and right 1 unit. Plot the point. Draw a line through the two points.
5-5 Inequalities Involving Absolute Value

68. \(-10 = 5y - 2x\)

**SOLUTION:**

Write the equation in slope-intercept form.

\[
-10 = 5y - 2x \\
-10 - 5y = 5y - 5y - 2x \\
-10 - 5y = -2x \\
-10 - 5y + 10 = -2x + 10 \\
-5y = -2x + 10 \\
\frac{-5y}{-5} = \frac{-2x + 10}{-5} \\
y = \frac{2}{5}x - 2
\]

The slope is \(\frac{2}{5}\) and the y-intercept is \(-2\). To graph the equation, plot the y-intercept \((0, -2)\). Then move up 2 units and right 5 units. Plot the point. Draw a line through the two points.