5-4 Solving Compound Inequalities

Solve each compound inequality. Then graph the solution set.
1. $4 \leq p - 8$ and $p - 14 \leq 2$

**SOLUTION:**

\[
\begin{align*}
4 & \leq p - 8 & & \text{and} & & p - 14 & \leq 2 \\
4 + 8 & \leq p - 8 + 8 & & & & p - 14 + 14 & \leq 2 + 14 \\
12 & \leq p & & & & p & \leq 16
\end{align*}
\]

The solution set is \( \{ p \mid 12 \leq p \leq 16 \} \).

To graph the solution set, graph $12 \leq p$ and graph $p \leq 16$. Then find the intersection.

2. $r + 6 < -8$ or $r - 3 > -10$

**SOLUTION:**

\[
\begin{align*}
r + 6 & < -8 & & \text{or} & & r - 3 & > -10 \\
r + 6 - 6 & < -8 - 6 & & & & r - 3 + 3 & > -10 + 3 \\
r & < -14 & & & & r & > -7
\end{align*}
\]

The solution set is \( \{ r \mid r < -14 \text{ or } r > -7 \} \).

Notice that the graphs do not intersect. To graph the solution set, graph $r < -14$ and graph $r > -7$. Then find the union.

3. $4a + 7 \geq 31$ or $a > 5$

**SOLUTION:**

\[
\begin{align*}
4a + 7 & \geq 31 & & \text{or} & & a & > 5 \\
4a + 7 - 7 & \geq 31 - 7 & & & & 4 & \geq 24 \\
4a & \geq 24 & & & & 4 & \geq 24 \\
\frac{4a}{4} & \geq \frac{24}{4} & & & & a & \geq 6
\end{align*}
\]

Notice that the two inequalities overlap at $a > 5$, so the solution set is \( \{ a \mid a > 5 \} \).

To graph the solution set, graph $a > 5$. 
5-4 Solving Compound Inequalities

4. $2 \leq g + 4 < 7$

**SOLUTION:**

\[
2 \leq g + 4 \quad \text{and} \quad g + 4 < 7
\]

\[
2 - 4 \leq g + 4 - 4 \quad g + 4 - 4 < 7 - 4
\]

\[
-2 \leq g \quad g < 3
\]

The solution set is \{ $g \mid -2 \leq g < 3$ \}.

To graph the solution set, graph $-2 \leq g$ and graph $g < 3$. Then find the intersection.

5. **BIKES** The recommended air pressure for the tires of a mountain bike is at least 35 pounds per square inch (psi), but no more than 80 pounds per square inch. If a bike’s tires have 24 pounds per square inch, what is the recommended range of air that should be put into the tires?

**SOLUTION:**

Let $x$ be the air pressure. The phrase *at least* means the same as greater than or equal to. The phrase *no more than* means the same as less than or equal to. The word *but* indicates that the problem represents an intersection.

\[
x \geq 35 - 24 \quad \text{and} \quad x \leq 80 - 24
\]

\[
x \geq 11 \quad \text{and} \quad x \leq 56
\]

So, an inequality that represents the range of recommended air pressure for tires is $11 \text{ psi} \leq x \leq 56 \text{ psi}$.

**Solve each compound inequality. Then graph the solution set.**

6. $f - 6 < 5$ and $f - 4 \geq 2$

**SOLUTION:**

\[
f - 6 < 5 \quad \text{and} \quad f - 4 \geq 2
\]

\[
f - 6 + 6 < 5 + 6 \quad f - 4 + 4 \geq 2 + 4
\]

\[
f < 11 \quad f \geq 6
\]

The solution set is \{ $f \mid 6 \leq f < 11$ \}.

To graph the solution set, graph $6 \leq f$ and graph $f < 11$. Then find the intersection.

7. $n + 2 \leq -5$ and $n + 6 \geq -6$

**SOLUTION:**

\[
n + 2 \leq -5 \quad \text{and} \quad n + 6 \geq -6
\]

\[
n + 2 - 2 \leq -5 - 2 \quad n + 6 - 6 \geq -6 - 6
\]

\[
n \leq -7 \quad n \geq -12
\]

The solution set is \{ $n \mid -12 \leq n \leq -7$ \}.

To graph the solution set, graph $-12 \leq n$ and graph $n \leq -7$. Then find the intersection.
5-4 Solving Compound Inequalities

8. \( y - 1 \geq 7 \) or \( y + 3 < -1 \)

**SOLUTION:**

\[
\begin{align*}
    y - 1 & \geq 7 & \text{or} & \quad y + 3 & < -1 \\
    y - 1 + 1 & \geq 7 + 1 & \text{or} & \quad y + 3 - 3 & < -1 - 3 \\
    y & \geq 8 & \text{or} & \quad y & < -4
\end{align*}
\]

The solution set is \{y \mid y \geq 8 \text{ or } y < -4\}. Notice that the graphs do not intersect. To graph the solution set, graph \( y \geq 8 \) and graph \( y < -4 \). Then find the union.

9. \( t + 14 \geq 15 \) or \( t - 9 < -10 \)

**SOLUTION:**

\[
\begin{align*}
    t + 14 & \geq 15 & \text{or} & \quad t - 9 & < -10 \\
    t + 14 - 14 & \geq 15 - 14 & \text{or} & \quad t - 9 + 9 & < -10 + 9 \\
    t & \geq 1 & \text{or} & \quad t & < -1
\end{align*}
\]

The solution set is \( \{t \mid t \geq 1 \text{ or } t < -1\} \). Notice that the graphs do not intersect. To graph the solution set, graph \( t \geq 1 \) and \( t < -1 \). Then find the union.

10. \( -5 < 3p + 7 \leq 22 \)

**SOLUTION:**

\[
\begin{align*}
    -5 & < 3p + 7 & \text{and} & \quad 3p + 7 & \leq 22\\
    -5 - 7 & < 3p + 7 - 7 & \text{and} & \quad 3p + 7 - 7 & \leq 22 - 7\\
    -12 & < 3p & \text{and} & \quad 3p & \leq 15\\
    -12 & < 3p & \leq 15 & \text{and} & \quad 3p & \leq 15\\
    -4 & < p & \leq 5 & \text{and} & \quad p & \leq 5
\end{align*}
\]

The solution set is \( \{p \mid -4 < p \leq 5\} \). To graph the solution set, graph \( -4 < p \) and graph \( p \leq 5 \). Then find the intersection.
5-4 Solving Compound Inequalities

11. \(-3 \leq 7c + 4 < 18\)

**SOLUTION:**

\[
\begin{array}{c|c|c}
-3 \leq 7c + 4 & 7c + 4 < 18 \\
-3 - 4 \leq 7c + 4 - 4 & 7c + 4 - 18 < 18 - 4 \\
-7 \leq 7c & 7c < 14 \\
\frac{-7}{7} \leq \frac{7c}{7} & \frac{7c}{7} < \frac{14}{7} \\
-1 \leq c & c < 2 \\
\end{array}
\]

The solution set is \( \{c \mid -1 \leq c < 2\} \).

To graph the solution set, graph \(-1 \leq c\) and graph \(c < 2\). Then find the intersection.

12. \(5h - 4 \geq 6\) and \(7h + 11 < 32\)

**SOLUTION:**

\[
\begin{array}{c|c|c}
5h - 4 \geq 6 & 7h + 11 < 32 \\
5h - 4 + 4 \geq 6 + 4 & 7h + 11 - 11 < 32 - 11 \\
5h \geq 10 & 7h < 21 \\
\frac{5h}{5} \geq \frac{10}{5} & \frac{7h}{7} < \frac{21}{7} \\
h \geq 2 & h < 3 \\
\end{array}
\]

The solution set is \( \{h \mid 2 \leq h < 3\} \).

To graph the solution set, graph \(2 \leq h\) and graph \(h < 3\). Then find the intersection.

13. \(22 \geq 4m - 2\) or \(5 - 3m \leq -13\)

**SOLUTION:**

\[
\begin{array}{c|c|c}
22 \geq 4m - 2 & 5 - 3m \leq -13 \\
22 + 2 \geq 4m - 2 + 2 & 5 - 3m - 13 \leq -13 - 5 \\
24 \geq 4m & -3m \leq -18 \\
\frac{24}{4} \geq \frac{4m}{4} & \frac{-3m}{-3} \geq \frac{-18}{-3} \\
6 \geq m & m \geq 6 \\
\end{array}
\]

Notice that the two inequalities overlap and all real numbers are solutions.

The solution set is \( \{m \mid m \text{ is a real number} \} \).

To graph the solution set, graph all points.
14. \(-4a + 13 \geq 29\) and \(10 < 6a - 14\)

**SOLUTION:**

\[
\begin{align*}
-4a + 13 & \geq 29 \\
-4a + 13 - 13 & \geq 29 - 13 \\
-4a & \geq 16 \\
\frac{-4a}{-4} & \leq \frac{16}{-4} \\
a & \leq -4 \\
\end{align*}
\]

\[
\begin{align*}
10 & < 6a - 14 \\
10 + 14 & < 6a - 14 + 14 \\
24 & < 6a \\
\frac{24}{6} & < \frac{6a}{6} \\
4 & < a \\
\end{align*}
\]

Notice that the two inequalities do not overlap.
So, the solution set is empty, \(\varnothing\).
The graph is also empty.

---

15. \(-y + 5 \geq 9\) or \(3y + 4 < -5\)

**SOLUTION:**

\[
\begin{align*}
-y + 5 & \geq 9 \\
-y + 5 - 5 & \geq 9 - 5 \\
y & \leq 4 \\
\frac{-y}{-1} & \leq \frac{4}{-1} \\
y & \leq -4 \\
\end{align*}
\]

\[
\begin{align*}
3y + 4 & < -5 \\
3y + 4 - 4 & < -5 - 4 \\
3y & < -9 \\
\frac{3y}{3} & < \frac{-9}{3} \\
y & < -3 \\
\end{align*}
\]

Notice that the two inequalities overlap at \(y < -3\), so the solution set is \(\{y \mid y < -3\}\).
To graph the solution set, graph \(y < -3\).

---

16. **SPEED**
The posted speed limit on an interstate highway is shown. Write an inequality that represents the sign.
Graph the inequality.

**SOLUTION:**

Sample answer: Let \(r\) = rate of speed. The lowest speed you can go is 40 mph, while the highest speed is 70 mph.
Therefore, the inequality is \(40 \leq r \leq 70\).
5-4 Solving Compound Inequalities

17. **NUMBER THEORY**  Find all sets of two consecutive positive odd integers with a sum that is at least 8 and less than 24.

**SOLUTION:**
Sample answer: Let \( x \) = the smaller of two consecutive odd numbers, then \( 8 \leq 2x + 2 \leq 24 \).

\[
\begin{align*}
8 & \leq 2x + 2 \leq 24 \\
8 - 2 & \leq 2x \leq 24 - 2 \\
6 & \leq 2x \leq 22 \\
\frac{6}{2} & \leq x \leq \frac{22}{2} \\
3 & \leq x \leq 11 \\
\end{align*}
\]

List every combination of numbers in which the smaller number is \( 3 \leq x \leq 11 \).
3, 5; 5, 7; 7, 9; 9, 11; 11, 13

**Write a compound inequality for each graph.**

18. \(-2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4\)

**SOLUTION:**
This graph represents an intersection. Both endpoints are closed circles which include the endpoints. The compound inequality is \(-1 \leq x \leq 4\).

19. \(-4 \quad -3 \quad -2 \quad 0 \quad 1 \quad 2\)

**SOLUTION:**
This graph represents an intersection. The left endpoint is an open circle which represents greater than. The right endpoint is a closed circle which represents less than or equal to. The compound inequality is \(-3 < x \leq 2\).

20. \(-1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4\)

**SOLUTION:**
The graphs do not intersect, so it represents a union. The endpoint on the left is an open endpoint, which represents less than. The endpoint on the right is a closed endpoint, which represents greater than or equal to. The inequalities are \( x < 0 \) or \( x \geq 3 \).

21. \(-6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0\)

**SOLUTION:**
The graphs do not intersect, so it represents a union. Both endpoints are open, so the endpoints are not included. The inequalities are \( x < -4 \) or \( x \geq -3 \).

22. \(0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7\)

**SOLUTION:**
The graphs do not intersect, so it represents a union. The endpoint on the left is a closed endpoint, which represents less than or equal to. The endpoint on the right is only a point. The inequalities are \( x \leq 3 \) or \( x \geq 6 \).
5-4 Solving Compound Inequalities

23.

-4 -3 -2 -1 0 1 2

SOLUTION:
The graphs do not intersect, so it represents a union. The endpoint on the right is an open endpoint, which represents greater than. The endpoint on the left is only a point. The inequalities are \( x \leq -3 \) or \( x > 0 \).

Solve each compound inequality. Then graph the solution set.
24. \( 3b + 2 < 5b - 6 \leq 2b + 9 \)

SOLUTION:

\[
\begin{align*}
3b + 2 &< 5b - 6 \\
3b - 3b + 2 &< 5b - 3b - 6 \\
2 &< 2b - 6 \\
2 + 6 &< 2b - 6 + 6 \\
8 &< 2b \\
\frac{8}{2} &< \frac{2b}{2} \\
4 &< b
\end{align*}
\]

\[
\begin{align*}
5b - 6 &\leq 2b + 9 \\
5b - 2b - 6 &\leq 2b - 2b + 9 \\
3b - 6 + 6 &\leq 9 + 6 \\
3b &\leq 15 \\
\frac{3b}{3} &\leq \frac{15}{3} \\
b &\leq 5
\end{align*}
\]

The solution set is \( \{ b \mid 4 < b \leq 5 \} \).
To graph the solution set, graph \( 4 < b \) and graph \( b \leq 5 \). Then find the intersection.

25. \( -2a + 3 \geq 6a - 1 > 3a - 10 \)

SOLUTION:

\[
\begin{align*}
-2a + 3 &\geq 6a - 1 \\
-2a + 2a + 3 &\geq 6a + 2a - 1 \\
3 + 1 &\geq 8a - 1 + 1 \\
4 &\geq 8a \\
\frac{4}{8} &\geq \frac{8a}{8} \\
\frac{1}{2} &\geq a
\end{align*}
\]

\[
\begin{align*}
6a - 1 &> 3a - 10 \\
6a - 6a - 1 &> 3a - 6a - 10 \\
-1 + 10 &> -3a - 10 + 10 \\
9 &> -3a \\
\frac{9}{-3} &> \frac{-3a}{-3} \\
-3 &> a
\end{align*}
\]

The solution set is \( \{ a \mid -3 < a \leq \frac{1}{2} \} \).
To graph the solution set, graph \( \frac{1}{2} \geq a \) and graph \( -3 > a \). Then find the intersection.
5-4 Solving Compound Inequalities

26. $10m - 7 < 17m$ or $-6m > 36$

**SOLUTION:**

\[
\begin{align*}
10m - 7 &< 17m & 10m &- 10m &-7 < 17m &-10m \\
& & & &-7 &< 7m \\
& & & &7 &> m \\
& & & &\frac{-7}{7} &> \frac{7}{7} \\
& & & &-1 &< m \\
\end{align*}
\]

The solution set is \{\(m \mid m < -6 \text{ or } m > -1\}\}.

Notice that the graphs do not intersect. To graph the solution set, graph \(m < -6\) and graph \(m > -1\). Then find the union.

```
-7 -6 -5 -4 -3 -2 -1 0 1
```

27. $5n - 1 < -16$ or $-3n - 1 < 8$

**SOLUTION:**

\[
\begin{align*}
5n - 1 &< -16 & -3n - 1 &< 8 \\
5n - 1 &+ 1 < -16 &+ 1 &-3n - 1 &+ 8 &+ 1 \\
5n &< -15 &-3n &< 9 \\
\frac{5n}{5} &< \frac{-15}{5} &\frac{-3n}{-3} &> \frac{9}{-3} \\
n &< -3 &n &> -3
\end{align*}
\]

Notice that the two inequalities overlap, but the point $-3$ is not included.

The solution set is \{\(n \mid n < -3 \text{ or } n > -3\}\}.

To graph the solution set, graph all points except for $-3$.

```
-8 -7 -6 -5 -4 -3 -2 -1 0 1 2
```
28. **COUPON** Juanita has a coupon for 10% off any digital camera at a local electronics store. She is looking at digital cameras that range in price from $100 to $250.

   a. How much are the cameras after the coupon is used?
   b. If the tax amount is 6.5%, how much should Juanita expect to spend?

**SOLUTION:**

   a. The least expensive camera is $100.
      The 10% coupon is worth $100 \times 0.10 or $10.
      After the coupon is used, the least expensive camera is $100 - $10 or $90.
      The most expensive camera is $250.
      The 10% coupon is worth $250 \times 0.10 or $25.
      After the coupon is used, the most expensive camera is $250 - $25 or $225.

      The cameras are between $90 and $225 inclusive.

   b. The most expensive camera is $90 after the coupon, so add 6.5% tax.

      \[ 90 \times 0.065 = 5.85 \]
      \[ 90 + 5.85 = 95.85 \]

      So, the least expensive camera will cost $95.85.
      The most expensive camera is $225 after the coupon, so add 6.5% tax.

      \[ 225 \times 0.065 = 14.63 \]
      \[ 225 + 14.63 = 239.63 \]

      So the most expensive camera will cost $239.63.
      Juanita should expect to spend between $95.85 and $239.63 inclusive.
5-4 Solving Compound Inequalities

Define a variable, write an inequality, and solve each problem. Then check your solution.

29. Eight less than a number is no more than 14 and no less than 5.

**SOLUTION:**
Let \( n \) be the number.

\[
\begin{align*}
5 & \leq n - 8 \leq 14 \\
5 + 8 & \leq n - 8 + 8 \leq 14 + 8 \\
13 & \leq n \leq 22
\end{align*}
\]

The solution set is \( \{ n \mid 13 \leq n \leq 22 \} \).
To check this answer, substitute a number greater than or equal to 13 and less than or equal to 22 into the original inequality. Let \( n = 15 \).

\[
\begin{align*}
5 & \leq n - 8 \leq 14 \\
5 & \leq 15 - 8 \leq 14 \\
5 & \leq 7 \leq 14
\end{align*}
\]

So, the solution checks.

30. The sum of 3 times a number and 4 is between \(-8\) and 10.

**SOLUTION:**
Let \( n \) be the number.

\[
\begin{align*}
-8 & < 3n + 4 < 10 \\
-8 - 4 & < 3n + 4 - 4 < 10 - 4 \\
-12 & < 3n < 6 \\
\frac{-12}{3} & < \frac{3n}{3} < \frac{6}{3} \\
-4 & < n < 2
\end{align*}
\]

The solution set is \( \{ n \mid -4 < n < 2 \} \).
To check this answer, substitute a number greater than \(-4\) and less than 2 into the original inequality. Let \( n = 0 \).

\[
\begin{align*}
-8 & < 3n + 4 < 10 \\
-8 & < 3(0) + 4 < 10 \\
-8 & < 0 + 4 < 10 \\
-8 & < 4 < 10
\end{align*}
\]

So, the solution checks.
5-4 Solving Compound Inequalities

31. The product of –5 and a number is greater than 35 or less than 10.

**SOLUTION:**
Let \( n \) = the number.

\[
\begin{align*}
-5n &> 35 \\
&\quad \text{or} \quad -5n < 10 \\
\frac{-5n}{-5} &< \frac{35}{-5} \\
\frac{-5n}{-5} &> \frac{10}{-5} \\
-5n &< -7 \\
-5 &> -5 \\
n &> -7 \\
n &< -2
\end{align*}
\]

The solution set is \( \{ n \mid n < -7 \text{ or } n > -2 \} \).
To check this answer, substitute a number less than –7 into the original inequality. Let \( n = -8 \).

\[
\begin{align*}
-5n &> 35 \\
-5(-8) &> 35 \\
40 &> 35
\end{align*}
\]

So, the solution checks.
Now check the second inequality. To check this, substitute a number that is greater than –2 into the original inequality. Let \( n = 0 \),

\[
\begin{align*}
-5n &< 10 \\
-5(0) &< 10 \\
0 &< 10
\end{align*}
\]

So, the solution checks.


5-4 Solving Compound Inequalities

32. One half a number is greater than 0 and less than or equal to 1.

**SOLUTION:**

Let \( n \) = the number.

\[
\begin{align*}
0 < \frac{1}{2}n & \leq 1 \\
2(0) < 2 \left( \frac{1}{2}n \right) & \leq 2(1) \\
0 < n & \leq 2
\end{align*}
\]

The solution set is \( \{ n \mid 0 < n \leq 2 \} \).

To check this answer, substitute a number greater than 0 and less than or equal to 2 into the original inequality. Let \( n = 1 \).

\[
\begin{align*}
0 < \frac{1}{2}n & \leq 1 \\
0 < \frac{1}{2}(1) & \leq 1 \\
0 < \frac{1}{2} & \leq 1
\end{align*}
\]

So, the solution checks.

33. **SNakes** Most snakes live where the temperature ranges from 75°F to 90°F inclusive. Write an inequality to represent temperatures where snakes will not thrive.

**SOLUTION:**

Snakes can live in temperatures between 75 and 90. This means that snakes do not typically live in temperatures lower than 75 or higher than 90. Let \( t \) represent the temperatures. So, the inequalities can be written as \( t < 75 \) or \( t > 90 \).

34. **Fundraising** Yumas is selling gift cards to raise money for a class trip. He can earn prizes depending on how many cards he sells. So far, he has sold 34 cards. How many more does he need to sell to earn a prize in category 4?

<table>
<thead>
<tr>
<th>Cards</th>
<th>Prize</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-15</td>
<td>1</td>
</tr>
<tr>
<td>16-30</td>
<td>2</td>
</tr>
<tr>
<td>31-45</td>
<td>3</td>
</tr>
<tr>
<td>46-60</td>
<td>4</td>
</tr>
<tr>
<td>+61</td>
<td>5</td>
</tr>
</tbody>
</table>

**SOLUTION:**

Yumas has sold 34 cards. The lowest number of cards he can sell to get a category 4 prize is 46, so he needs to sell at least 12 more cards. The maximum number of cards he can sell and still get a category 4 prize is 60, which means he needs to sell at most 26 cards. This means that he needs to sell between 12 and 26 inclusive.
5-4 Solving Compound Inequalities

35. **TURTLES** Atlantic sea turtle eggs that incubate below 23°C or above 33°C rarely hatch. Write the temperature requirements in two ways: as a pair of simple inequalities, and as a compound inequality.

**SOLUTION:**
Let \( t \) represent the temperature.
“cannot be below 23” is represented by \( 23 \leq t \).
“cannot be above 33” is represented by \( t \leq 33 \).
This compound inequality can be expressed in two ways:
\[
23 \leq t \leq 33
\]

36. **GEOMETRY** The **Triangle Inequality Theorem** states that the sum of the measures of any two sides of a triangle is greater than the measure of the third side.

a. Write and solve three inequalities to express the relationships among the measures of the sides of the triangle shown.

b. What are four possible lengths for the third side of the triangle?

c. Write a compound inequality for the possible values of \( x \).

**SOLUTION:**

a. 1st inequality: \( x + 9 > 4 \)
\[
x + 9 - 9 > 4 - 9
\]
\[
x > -5
\]
2nd inequality: \( x + 4 > 9 \)
\[
x + 4 - 4 > 9 - 4
\]
\[
x > 5
\]
3rd inequality: \( 4 + 9 > x \)
\[
13 > x
\]

b. Sample answer: Every side of a triangle must be positive, so the inequality \( x > -5 \) can be disregarded. The other two inequalities show that the third side of the triangle must be greater than 5, but less than 13. Four possibilities are 6, 9, 10, and 11.

c. The first inequality states that \( x \) must be greater than \(-5\); however a length may not be a negative number; therefore \( x \) must be greater than 0. The second inequality states that \( x \) must be greater than 5, which overlaps with the first inequality, while the third inequality states that \( x \) must be less than 13. Therefore, the compound inequality is \( 5 < x < 13 \).
5-4 Solving Compound Inequalities

37. HURRICANES  The Saffir–Simpson Hurricane Scale rates hurricanes on a scale from 1 to 5 based on their wind speed.

a. Write a compound inequality for the wind speeds of a category 3 and a category 4 hurricane.

b. What is the intersection of the two graphs of the inequalities you found in part a?

SOLUTION:

a. Let \( x \) represent the wind speed.
For a category 3: \( 111 \leq x \leq 130 \)
For a category 4: \( 131 \leq x \leq 155 \)

b. The union of the two graphs is where either of the graphs are. So the solution is \( \{ x \mid 111 \leq x \leq 155 \} \). The intersection is where it overlaps. However, these graphs do not overlap, so it is an empty set or \( \emptyset \).

38. MULTIPLE REPRESENTATIONS  In this problem, you will investigate measurements. The absolute error of a measurement is equal to one half the unit of measure. The relative error of a measure is the ratio of the absolute error to the expected measure.

a. TABULAR  Copy and complete the table.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Absolute Error</th>
<th>Relative Error</th>
</tr>
</thead>
</table>
| 14.3 cm | \( \frac{1}{2}(0.1) = 0.05 \text{ cm} \) | \[
\frac{\text{absolute error}}{\text{expected measure}} = \frac{0.05 \text{ cm}}{14.3 \text{ cm}} \approx 0.0035 \text{ or } 0.4\%
\] |
| 1.85 cm | | |
| 61.2 cm | | |
| 237 cm | | |

b. ANALYTICAL  You measured a length of 12.8 centimeters. Compute the absolute error and then write the range of possible measures.

c. LOGICAL  To what precision would you have to measure a length in centimeters to have an absolute error of less than 0.05 centimeters?

d. ANALYTICAL  To find the relative error of an area or volume calculation, add the relative errors of each linear measure. If the measures of the sides of a rectangular box are 6.5 centimeters, 7.2 centimeters, and 10.25 centimeters, what is the relative error of the volume of the box?

SOLUTION:
5-4 Solving Compound Inequalities

b. The absolute error is one half the unit measure, $\frac{1}{2}(0.1) = 0.05$ cm; The smallest measurement would be the length minus the absolute error, $12.8 - 0.05 = 12.75$. The largest measurement would be the length plus the absolute error, $12.8 + 0.05 = 12.85$.

c. 0.05 is the absolute error for $0.1 \div 2 = 0.05$. Since it has to be less than 0.1, the precision must be in the hundredths or to the nearest hundredths place

d. The relative error of the first linear measure is $\frac{0.05}{6.5} \approx 0.0076923077$.

The relative error of the second linear measure is $\frac{0.05}{7.2} \approx 0.0069444444$.

The relative error of the third linear measure is $\frac{0.005}{10.25} \approx 0.000487804878$.

To find the relative error of the volume, add these together: $0.0077 + 0.0069 + 0.0004 = 0.015$

39. ERROR ANALYSIS Chloe and Jonas are solving the $3 < 2x - 5 < 7$. Is either of them correct? Explain your reasoning.

**SOLUTION:**
Neither of them are correct. Chloe did not add 5 to 3, and Jonas did not add 5 to 7. They each only added the 5 to one side of the compound inequality, not both.
5-4 Solving Compound Inequalities

40. CHALLENGE Solve each inequality for \( x \). Assume \( a \) is constant and \( a > 0 \).

a. \(-3 < ax + 1 \leq 5\)

b. \(\frac{-1}{a}x + 6 < 1 \text{ or } 2 - ax > 8\)

SOLUTION:

a. \(-3 < ax + 1 \leq 5\)

\[
-3 < ax + 1 \leq 5 \\
-3 - 1 < ax + 1 - 1 \leq 5 - 1 \\
-4 < ax \leq 4 \\
\frac{-4}{a} < x \leq \frac{4}{a}
\]

b.

\[\frac{-1}{a}x + 6 < 1\]
\[
\frac{-1}{a}x + 6 - 6 < 1 - 6 \\
\frac{-1}{a}x < -5 \\
(-a)\left(\frac{-1}{a}x\right) > (-a)(-5) \\
x > 5a
\]

or

\[2 - ax > 8 \]
\[2 - ax - 2 > 8 - 2 \]
\[-ax > 6 \]
\[\frac{-ax}{-a} < \frac{6}{-a} \]
\[x < \frac{6}{-a}\]

41. OPEN ENDED Create an example of a compound inequality containing or that has infinitely many solutions.

SOLUTION:

Answers may vary. Sample answer: \( x \leq 4 \) or \( x \geq 4 \)

42. CHALLENGE Determine whether the following statement is always, sometimes, or never true. Explain. The graph of a compound inequality that involves an or statement is bounded on the left and right by two values of \( x \).

SOLUTION:

Sometimes; the graph of \( x > 2 \) or \( x < 5 \) includes the entire number line.
43. **WRITING IN MATH**  Give an example of a compound inequality you might encounter at an amusement park. Does the example represent an intersection or a union?

**SOLUTION:**
Sample answer: The speed at which a roller coaster runs while staying on the track could represent a compound inequality that is an intersection.

44. What is the solution set of the inequality \(-7 < x + 2 < 4\)?

- **A** \(x \mid -5 < x < 6\)
- **B** \(x \mid -5 < x < 2\)
- **C** \(x \mid -9 < x < 2\)
- **D** \(x \mid -9 < x < 6\)

**SOLUTION:**
\[-7 < x + 2 < 4\]
\[-7 - 2 < x + 2 - 2 < 4 - 2\]
\[-9 < x < 2\]
So, the correct choice is C.

45. **GEOMETRY**  What is the surface area of the rectangular solid?

![](image)

- **F** \(249.6 \text{ cm}^2\)
- **G** \(278.4 \text{ cm}^2\)
- **H** \(313.6 \text{ cm}^2\)
- **J** \(371.2 \text{ cm}^2\)

**SOLUTION:**
The surface area of the rectangular solid can be calculated by finding the area of each of the sides.

\[A = lh\]
\[(8)(5.8)\]
\[= 46.4\]

There are 4 sides with these dimensions, so the surface area of the four sides is \(4(46.4) = 185.6 \text{ cm}^2\). Another side has an area of

\[A = lw\]
\[(8)(8)\]
\[= 64\]

There are two sides with these dimensions, so the surface area of the two sides is \(2(64) = 128 \text{ cm}^2\). The total surface area is \(185.6 + 128 = 313.6 \text{ cm}^2\). So, the correct choice is H.
5-4 Solving Compound Inequalities

46. **GRIDDED RESPONSE** What is the next term in the sequence?

\[
\begin{array}{c}
13 & 18 & 23 & 28 & 33 \\
2 & 3 & 4 & 5 & 6
\end{array}
\]

**SOLUTION:**

The pattern is to add 5 to the numerator and add 3 to the denominator. So, the next term should be \(\frac{33 + 5}{14 + 3} = \frac{38}{17}\).

47. After paying a $15 membership fee, members of a video club can rent movies for $2. Nonmembers can rent movies for $4. What is the least number of movies which must be rented for it to be less expensive for members?

A 9  
B 8  
C 7  
D 6

**SOLUTION:**

Let \(m\) represent the number of movies.

\[
\begin{align*}
15 + 2m &< 4m \\
15 + 2m - 2m &< 4m - 2m \\
15 &< 2m \\
\frac{15}{2} &< \frac{2m}{2} \\
7.5 &< m
\end{align*}
\]

This means that they must rent more than 7.5 movies, so choices C and D can be eliminated. The question asks for the least number of movies that can be rented, so the correct choice is B.

48. **BABYSITTING** Marilyn earns $150 per month delivering newspapers plus $7 an hour babysitting. If she wants to earn at least $300 this month, how many hours will she have to babysit?

**SOLUTION:**

Let \(h\) represent the number of hours that Marilyn must babysit.

\[
\begin{align*}
150 + 7h &\geq 300 \\
150 - 150 + 7h &\geq 300 - 150 \\
7h &\geq 150 \\
\frac{7h}{7} &\geq \frac{150}{7} \\
h &\geq 21.43
\end{align*}
\]

So, she will need to babysit at least 22 hours to earn $300 this month.
5-4 Solving Compound Inequalities

49. **MAGAZINES** Carlos has earned more than $260 selling magazine subscriptions. Each subscription was sold for $12. How many did Carlos sell?

**SOLUTION:**

Let \( m \) represent the number of magazine subscriptions.

\[
\begin{align*}
12m & > 260 \\
12 & > 260 \\
\frac{12m}{12} & > \frac{260}{12} \\
\quad m & > 21.67
\end{align*}
\]

So, Carlos sold at least 22 subscriptions.

50. **PUNCH** Raquel is mixing lemon–lime soda and a fruit juice blend that is 45% juice. If she uses 3 quarts of soda, how many quarts of fruit juice must be added to produce punch that is 30% juice?

**SOLUTION:**

Let \( x \) represent the number of quarts of fruit juice Raquel must add. Set up an equation where the amount of juice in the mixture is equal to the amount of juice in the soda plus the amount of juice in the blend.

\[
\text{Amount of juice} = \text{Amount of juice} \\
\text{Mixture} = \text{soda} + \text{blend} \\
0.30(3 + x) = 0(3) + 0.45(x) \\
0.90 + 0.30x = 0 + 0.45x \\
0.90 + 0.30x - 0.30x = 0.45x - 0.30x \\
0.90 = 0.15x \\
\frac{0.90}{0.15} = \frac{0.15x}{0.15} \\
6 = x
\]

So, Raquel needs to add 6 quarts of fruit juice.

**Solve each proportion. If necessary, round to the nearest hundredth.**

51. \( \frac{14}{x} = \frac{20}{8} \)

**SOLUTION:**

\[
\begin{align*}
\frac{14}{x} & = \frac{20}{8} \\
\frac{14}{x} & = \frac{5}{2} \\
x(\frac{5}{2}) & = 14 \\
5x & = 28 \\
\frac{5x}{5} & = \frac{28}{5} \\
x & = 5.6
\end{align*}
\]
5-4 Solving Compound Inequalities

52. \( \frac{0.47}{6} = \frac{1.41}{m} \)

**SOLUTION:**

\[ \frac{0.47}{6} = \frac{1.41}{m} \]

\[ 0.47(m) = 6(1.41) \]

\[ 0.47m = 8.46 \]

\[ \frac{0.47m}{0.47} = \frac{8.46}{0.47} \]

\[ m = 18 \]

53. \( \frac{16}{7} = \frac{9}{b} \)

**SOLUTION:**

\[ \frac{16}{7} = \frac{9}{b} \]

\[ 16(b) = 9(7) \]

\[ 16b = 63 \]

\[ \frac{16b}{16} = \frac{63}{16} \]

\[ b = 3.9375 \]

\[ b \approx 3.94 \]

54. \( \frac{2 + y}{5} = \frac{10}{3} \)

**SOLUTION:**

\[ \frac{2 + y}{5} = \frac{10}{3} \]

\[ 3(2 + y) = 10(5) \]

\[ 6 + 3y = 50 \]

\[ 6 - 6 + 3y = 50 - 6 \]

\[ 3y = 44 \]

\[ \frac{3y}{3} = \frac{44}{3} \]

\[ y = 14.67 \]
5-4 Solving Compound Inequalities

55. \[ \frac{8}{9} = \frac{2r-3}{4} \]

SOLUTION:
\[
\frac{8}{9} = \frac{2r - 3}{4} \\
(2r - 3)(9) = 4(8) \\
18r - 27 = 32 \\
18r = 59 \\
r = \frac{59}{18} \\
r = 3.28
\]

56. \[ \frac{6-2y}{8} = \frac{2}{18} \]

SOLUTION:
\[
\frac{6-2y}{8} = \frac{2}{18} \\
18(6-2y) = 2(8) \\
108 - 36y = 16 \\
108 - 108 - 36y = 16 - 108 \\
-36y = -92 \\
\frac{-36y}{-36} = \frac{-92}{-36} \\
y = 2.56
\]

Determine whether each relation is a function. Explain.

<table>
<thead>
<tr>
<th>Domain</th>
<th>2</th>
<th>6</th>
<th>10</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

SOLUTION:
In a function, there is exactly one output for every input. 2 is paired with 5, 6 is paired with 0, 10 is paired with 5, and 7 is paired with 0, so this relation is a function.

<table>
<thead>
<tr>
<th>Domain</th>
<th>-5</th>
<th>2</th>
<th>-3</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>-10</td>
<td>-7</td>
<td>-5</td>
<td>-3</td>
</tr>
</tbody>
</table>

SOLUTION:
In a function, there is exactly one output for every input. -5 is paired with -10, 2 is paired with -7, -3 is paired with -5, and 2 is paired with -3. Notice that 2 is paired with both -7 and -3. This relation is not a function.
5-4 Solving Compound Inequalities

59. \{(−4, 11), (−2, 7), (1, 3), (−4, −1)\}

**SOLUTION:**
In a function, there is exactly one output for every input. −4 is paired with 11, −2 is paired with 7, 1 is paired with 3, and −4 is paired with −1. Notice that −4 is paired with both 11 and −1. This relation is not a function.

60. \{(2, 7), (5, −3), (7, 6), (10, 7)\}

**SOLUTION:**
In a function, there is exactly one output for every input. 2 is paired with 7, 5 is paired with −3, 7 is paired with 6, and 10 is paired with 7. So, this relation is a function.

**Evaluate each expression.**

61. \(5 + (4 − 2^2)\)

**SOLUTION:**
\[
\begin{align*}
5 + (4 − 2^2) & \quad 2^2 = 4 \\
= 5 + (4 − 4) & \quad \text{Subtract.} \\
= 5 + 0 & \quad \text{Additive Identity} \\
= 5 & \\
\end{align*}
\]

62. \(\frac{3}{8} \left[8 + (7 − 4)\right]\)

**SOLUTION:**
\[
\begin{align*}
\frac{3}{8} \left[8 + (7 − 4)\right] & \quad \text{Subtract.} \\
= \frac{3}{8} \left[8 + 3\right] & \quad 8 + 3 = \frac{8}{3} \\
= \frac{3}{8} \cdot \frac{8}{3} & \quad \text{Multiplicative Inverse} \\
= 1 & \\
\end{align*}
\]
Solve each equation.

63. \(2(4 \cdot 9 - 3) + 5 \cdot \frac{1}{5}\)

**SOLUTION:**

\[
2(4 \cdot 9 - 3) + 5 \cdot \frac{1}{5}
\]

\[
= 2(36 - 3) + 5 \cdot \frac{1}{5}
\]

\[
= 2(33) + 5 \cdot \frac{1}{5}
\]

\[
= 66 + 5 \cdot \frac{1}{5}
\]

\[
= 66 + 1
\]

\[
= 67
\]

64. \(4p - 2 = -6\)

**SOLUTION:**

\[
4p - 2 = -6
\]

\[
4p - 2 + 2 = -6 + 2
\]

\[
4p = -4
\]

\[
4p = -4
\]

\[
4 = 4
\]

\[
p = -1
\]

65. \(18 = 5p + 3\)

**SOLUTION:**

\[
18 = 5p + 3
\]

\[
18 - 3 = 5p + 3 - 3
\]

\[
15 = 5p
\]

\[
\frac{15}{5} = \frac{5p}{5}
\]

\[
3 = p
\]
5-4 Solving Compound Inequalities

66. \( 9 = 1 + \frac{m}{7} \)

\textbf{SOLUTION:}

\[
9 = 1 + \frac{m}{7} \\
9 - 1 = 1 + \frac{m}{7} - 1 \\
8 = \frac{m}{7} \\
7(8) = 7 \left( \frac{m}{7} \right) \\
56 = m
\]

67. \( 1.5a - 8 = 11 \)

\textbf{SOLUTION:}

\[
1.5a - 8 = 11 \\
1.5a - 8 + 8 = 11 + 8 \\
1.5a = 19 \\
\frac{1.5a}{1.5} = \frac{19}{1.5} \\
a = 12 \frac{2}{3}
\]

68. \( 20 = -4c - 8 \)

\textbf{SOLUTION:}

\[
20 = -4c - 8 \\
20 + 8 = -4c - 8 + 8 \\
28 = -4c \\
\frac{28}{-4} = \frac{-4c}{-4} \\
-7 = c
\]

69. \( \frac{b + 4}{-2} = -17 \)

\textbf{SOLUTION:}

\[
\frac{b + 4}{-2} = -17 \\
-2 \left( \frac{b + 4}{-2} \right) = -2(-17) \\
b + 4 = 34 \\
b + 4 - 4 = 34 - 4 \\
b = 30
\]
5-4 Solving Compound Inequalities

70. \( \frac{n - 3}{8} = 20 \)

**SOLUTION:**
\[
\frac{n - 3}{8} = 20 \\
8 \left( \frac{n - 3}{8} \right) = 8(20) \\
n - 3 = 160 \\
n - 3 + 3 = 160 + 3 \\
n = 163
\]

71. \( 6y - 16 = 44 \)

**SOLUTION:**
\[
6y - 16 = 44 \\
6y - 16 + 16 = 44 + 16 \\
6y = 60 \\
\frac{6y}{6} = \frac{60}{6} \\
y = 10
\]

72. \( 130 = 11k + 9 \)

**SOLUTION:**
\[
130 = 11k + 9 \\
130 - 9 = 11k + 9 - 9 \\
121 = 11k \\
\frac{121}{11} = \frac{11k}{11} \\
11 = k
\]