3-5 Arithmetic Sequences as Linear Functions

**Determine whether each sequence is an arithmetic sequence. Write yes or no. Explain.**

1. 18, 16, 15, 13, …

**SOLUTION:**
An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference. To find the common difference, subtract two consecutive numbers in the sequence.

\[
\begin{align*}
16 - 18 &= -2 \\
15 - 16 &= -1 \\
13 - 15 &= -2 \\
\end{align*}
\]

The difference between terms is not constant. Therefore, it is not an arithmetic sequence.

2. 4, 9, 14, 19, …

**SOLUTION:**
An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference. To find the common difference, subtract two consecutive numbers in the sequence.

\[
\begin{align*}
9 - 4 &= 5 \\
14 - 9 &= 5 \\
19 - 14 &= 5 \\
\end{align*}
\]

The difference between terms is constant, so the sequence is an arithmetic sequence.

The common difference is 5.

**Find the next three terms of each arithmetic sequence.**

3. 12, 9, 6, 3, …

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

\[
6 - 9 = -3.
\]

The common difference between terms is $-3$. So, to find the next term, subtract 3 from the last term. To find the next term, subtract 3 from the resulting number, and so on.

\[
\begin{align*}
3 - 3 &= 0 \\
0 - 3 &= -3 \\
-3 - 3 &= -6 \\
\end{align*}
\]

So, the next three terms of this arithmetic sequence are 0, $-3$, $-6$. 
3-5 Arithmetic Sequences as Linear Functions

4. –2, 2, 6, 10, …

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

\[2 - (-2) = 4\]

The common difference between terms is 4. So, to find the next term, add 4 to the last term. To find the next term, add 4 to the resulting number, and so on.

\[10 + 4 = 14\]
\[14 + 4 = 18\]
\[18 + 4 = 22\]

So, the next three terms of this arithmetic sequence are 14, 18, 22.

**Write an equation for the \(n\)th term of each arithmetic sequence. Then graph the first five terms of the sequence.**

5. 15, 13, 11, 9, …

**SOLUTION:**
Find the common difference.

\[13 - 15 = -2\]

Write the equation for the \(n\)th term of an arithmetic sequence using the first term 15 and common difference \(-2\).

\[a_n = a_1 + (n - 1)d\]
\[= 15 + (n - 1)(-2)\]
\[= 15 - 2n + 2\]
\[= 17 - 2n\]

The points to graph are represented by \((n, a_n)\). So, the first four points are \((1, 15)\); \((2, 13)\); \((3, 11)\); \((4, 9)\). To find the fifth point, substitute 5 for \(n\) in the equation and evaluate for \(a_n\).

\[a_n = 17 - 2n\]
\[= 17 - 2(5)\]
\[= 17 - 10\]
\[= 7\]

The fifth point is \((5, 7)\).
6. \(-1, -0.5, 0, 0.5, \ldots\)

**SOLUTION:**
Subtract the 1st term from the 2nd to find the common difference.
\[-0.5 - (-1) = 0.5\]

Write the equation for the \(n\)th term of an arithmetic sequence using the first term \(-1\) and common difference 0.5.

\[
a_n = a_1 + (n - 1)d
\]
\[
= -1 + (n - 1)0.5
\]
\[
= -1 + 0.5n - 0.5
\]
\[
= 0.5n - 1.5
\]

The points to graph are represented by \((n, a_n)\). So, the first four points are \((1, -1)\); \((2, -0.5)\); \((3, 0)\); \((4, 0.5)\). To find the fifth point, substitute 5 for \(n\) in the equation and evaluate for \(a_n\).

\[
a_n = 0.5n - 1.5
\]
\[
= 0.5(5) - 1.5
\]
\[
= 2.5 - 1.5
\]
\[
= 1
\]

The fifth point is \((5, 1)\).
7. **SAVINGS**

Kaia has $525 in a savings account. After one month she has $580 in the account. The next month the balance is $635. The balance after the third month is $690. Write a function to represent the arithmetic sequence. Then graph the function.

**SOLUTION:**

The arithmetic sequence is 525, 580, 635, 690, …

Subtract the 1st term from the 2nd to find the common difference.

\[ 580 - 525 = 55 \]

Write the equation for the \( n \)th term of an arithmetic sequence using the first term 580 (note: 525 = \( a_0 \)) and common difference 55.

\[
\begin{align*}
f_n &= a_1 + (n - 1)d \\
&= 580 + (n - 1)55 \\
&= 580 + 55n - 55 \\
&= 55n + 255
\end{align*}
\]

The points to graph are represented by \((n, f_n)\). So, the first four points are (0, 525); (1, 580); (2, 635); (3, 690).

---

**Determine whether each sequence is an arithmetic sequence. Write yes or no.**

**Explain.**

8. 

\(-3, 1, 5, 9, …\)

**SOLUTION:**

An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference. To find the common difference, subtract two consecutive numbers in the sequence.

\[
\begin{align*}
1 - (-3) &= 4 \\
5 - 1 &= 4 \\
9 - 5 &= 4
\end{align*}
\]

The difference between terms is constant, so the sequence is an arithmetic sequence.

The common difference is 4.
3-5 Arithmetic Sequences as Linear Functions

9. \( \frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, \ldots \)

**SOLUTION:**
An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference. To find the common difference, subtract two consecutive numbers in the sequence.

\[
\frac{3}{4} - \frac{1}{2} = \frac{1}{4} \\
\frac{5}{8} - \frac{3}{4} = -\frac{1}{8} \\
\frac{7}{10} - \frac{5}{8} = -\frac{3}{10}
\]

The difference between terms is not constant, so the sequence is not an arithmetic sequence.

10. \(-10, -7, -4, 1, \ldots\)

**SOLUTION:**
An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference. To find the common difference, subtract two consecutive numbers in the sequence.

\(71 - (-10) = 3\)
\(-4 - (-7) = 3\)
\(1 - (-4) = 5\)

The difference between terms is not constant, so the sequence is not an arithmetic sequence.

11. \(-12.3, -9.7, -7.1, -4.5, \ldots\)

**SOLUTION:**
An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference. To find the common difference, subtract two consecutive numbers in the sequence.

\(-9.7 - (-12.3) = 2.6\)
\(-7.1 - (-9.7) = 2.6\)
\(-4.5 - (-7.1) = 2.6\)

The difference between terms is constant, so the sequence is an arithmetic sequence.

The common difference is 2.6.
Find the next three terms of each arithmetic sequence.
12. 0.02, 1.08, 2.14, 3.2, …

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

1.08 − 0.02 = 1.06

The common difference between terms is 1.06. So, to find the next term, add 1.06 to the last term. To find the next term, add 1.06 to the resulting number, and so on.

3.2 + 1.06 = 4.26
4.26 + 1.06 = 5.32
5.32 + 1.06 = 6.38

So, the next three terms of this arithmetic sequence are 4.26, 5.32, 6.38.

13. 6, 12, 18, 24, …

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

12 − 6 = 6

The common difference between terms is 6. So, to find the next term, add 6 to the last term. To find the next term, add 6 to the resulting number, and so on.

24 + 6 = 30
30 + 6= 36
36 + 6 = 42

So, the next three terms of this arithmetic sequence are 30, 36, 42.

14. 21, 19, 17, 15, …

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

19 − 21 = −2

The common difference between terms is −2. So, to find the next term, subtract 2 from the last term. To find the next term, subtract 2 from the resulting number, and so on.

15 − 2 = 13
13 − 2 = 11
11 − 2 = 9

So, the next three terms of this arithmetic sequence are 13, 11, 9.
3-5 Arithmetic Sequences as Linear Functions

15. $-\frac{1}{2}, 0, \frac{1}{2}, 1, ...$

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

$0 - \left( -\frac{1}{2} \right) = \frac{1}{2}$

The common difference between terms is $\frac{1}{2}$. So, to find the next term, add $\frac{1}{2}$ to the last term. To find the next term, add $\frac{1}{2}$ to the resulting number, and so on.

\[
\begin{align*}
1 + \frac{1}{2} &= 1\frac{1}{2} \\
\frac{1}{2} + \frac{1}{2} &= 1 \\
2 + \frac{1}{2} &= 2\frac{1}{2}
\end{align*}
\]

So, the next three terms of this arithmetic sequence are $1\frac{1}{2}, 2, 2\frac{1}{2}$.

16. $2\frac{1}{3}, 2\frac{2}{3}, 3, 3\frac{1}{3}, ...$

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

$2\frac{2}{3} - 2\frac{1}{3} = \frac{1}{3}$

The common difference between terms is $\frac{1}{3}$. So, to find the next term, add $\frac{1}{3}$ to the last term. To find the next term, add $\frac{1}{3}$ to the resulting number, and so on.

\[
\begin{align*}
3 + \frac{1}{3} &= 3\frac{2}{3} \\
3\frac{2}{3} + \frac{1}{3} &= 4 \\
4 + \frac{1}{3} &= 4\frac{1}{3}
\end{align*}
\]

So, the next three terms of this arithmetic sequence are $3\frac{2}{3}, 4, 4\frac{1}{3}$. 
3-5 Arithmetic Sequences as Linear Functions

17. \( \frac{7}{12}, 1 \frac{1}{3}, 2 \frac{1}{12}, 2 \frac{5}{6}, \ldots \)

**SOLUTION:**
Find the common difference by subtracting two consecutive terms.

\[
\begin{align*}
1 \frac{1}{3} - \frac{7}{12} &= \frac{4}{3} - \frac{7}{12} \\
&= \frac{16}{12} - \frac{7}{12} \\
&= \frac{9}{12} \\
&= \frac{3}{4}
\end{align*}
\]

The common difference between term is \( \frac{3}{4} \). So, to find the next term, add \( \frac{3}{4} \) to the last term. To find the next term, add \( \frac{3}{4} \) to the resulting number, and so on.

\[
\begin{align*}
2 \frac{5}{6} + \frac{3}{4} &= \frac{17}{6} + \frac{3}{4} \\
&= \frac{34}{12} + \frac{9}{12} \\
&= \frac{43}{12} \\
&= 3 \frac{7}{12}
\end{align*}
\]

\[
\begin{align*}
3 \frac{7}{12} + \frac{3}{4} &= \frac{43}{12} + \frac{3}{4} \\
&= \frac{43}{12} + \frac{9}{12} \\
&= \frac{52}{12} \\
&= 4 \frac{4}{12} \\
&= 4 \frac{1}{3}
\end{align*}
\]

\[
\begin{align*}
4 \frac{1}{3} + \frac{3}{4} &= \frac{13}{3} + \frac{3}{4} \\
&= \frac{52}{12} + \frac{9}{12} \\
&= \frac{61}{12} \\
&= 5 \frac{1}{12}
\end{align*}
\]

So, the next three terms of this arithmetic sequence are \( \frac{7}{12}, \frac{1}{3}, \frac{5}{12} \).
3-5 Arithmetic Sequences as Linear Functions

Write an equation for the $n$th term of the arithmetic sequence. Then graph the first five terms in the sequence.

18. $-3, -8, -13, -18, \ldots$

**SOLUTION:**
Subtract the 1st term from the 2nd to find the common difference.

$$-8 - (-3) = -5$$

Write the equation for the $n$th term of an arithmetic sequence using the first term $-3$ and common difference $-5$.

$$a_n = a_1 + (n - 1)d$$

$$= -3 + (n - 1)(-5)$$

$$= -3 - 5n + 5$$

$$= -5n + 2$$

The points to graph are represented by $(n, a_n)$. So, the first four points are $(1, -3); (2, -8); (3, -13); (4, -18)$. To find the fifth point, substitute 5 for $n$ in the equation and evaluate for $a_n$.

$$a_n = -5n + 2$$

$$= -5(5) + 2$$

$$= -25 + 2$$

$$= -23$$

The fifth point is $(5, -23)$. 

![Graph of the arithmetic sequence](image)
3-5 Arithmetic Sequences as Linear Functions

19. –2, 3, 8, 13, …

**SOLUTION:**
Subtract the 1st term from the 2nd to find the common difference.

\[ 3 - (-2) = 5 \]

Write the equation for the \( n \)th term of an arithmetic sequence using the first term \(-2\) and common difference \(5\).

\[
\begin{align*}
  a_n &= a_1 + (n - 1)d \\
  &= -2 + (n - 1)5 \\
  &= -2 + 5n - 5 \\
  &= 5n - 7
\end{align*}
\]

The points to graph are represented by \((n, a_n)\). So, the first four points are \((1, -2); (2, 3); (3, 8); (4, 13)\). To find the fifth point, substitute \(5\) for \(n\) in the equation and evaluate for \(a_n\).

\[
\begin{align*}
  a_n &= 5n - 7 \\
  &= 5(5) - 7 \\
  &= 25 - 7 \\
  &= 18
\end{align*}
\]

The fifth point is \((5, 18)\).

---

\[(3, 0); (4, 0.5)\]
20. \(-11, -15, -19, -23, \ldots\)

**SOLUTION:**
Subtract the 1st term from the 2nd to find the common difference.

\[-15 - (-11) = -4\]

Write the equation for the \(n\)th term of an arithmetic sequence using the first term \(-11\) and common difference \(-4\).

\[
   a_n = a_1 + (n - 1)d
\]
\[
   = -11 + (n - 1)(-4)
\]
\[
   = -11 - 4n + 4
\]
\[
   = -4n - 7
\]

The points to graph are represented by \((n, a_n)\). So, the first four points are \((1, -11); (2, -15); (3, -19); (4, -23)\). To find the fifth point, substitute 5 for \(n\) in the equation and evaluate for \(a_n\):

\[
   a_n = -4n - 7
\]
\[
   = -4(5) - 7
\]
\[
   = -20 - 7
\]
\[
   = -27
\]

The fifth point is \((5, -27)\).
3-5 Arithmetic Sequences as Linear Functions

21. −0.75, −0.5, −0.25, 0, …

**SOLUTION:**
Subtract the 1st term from the 2nd to find the common difference.
−0.5 − (−0.75) = 0.25

Write the equation for the $n$th term of an arithmetic sequence using the first term $−0.75$ and common difference $0.25$.

$$a_n = a_1 + (n - 1)d$$

$$= −0.75 + (n - 1)0.25$$

$$= −0.75 + 0.25n − 0.25$$

$$= 0.25n − 1$$

The points to graph are represented by $(n, a_n)$. So, the first four points are $(1, −0.75); (2, −0.5); (3, −0.25); (4, 0)$. To find the fifth point, substitute 5 for $n$ in the equation and evaluate for $a_n$.

$$a_n = 0.25n − 1$$

$$= 0.25(5) − 1$$

$$= 1.25 − 1$$

$$= 0.25$$

The fifth point is $(5, 0.25)$. 

![Graph of the arithmetic sequence](image-url)
22. **AMUSEMENT PARKS**

Shiloh and her friends spent the day at an amusement park. In the first hour, they rode two rides. After 2 hours, they had ridden 4 rides. They had ridden 6 rides after 3 hours.

a. Write a function to represent the arithmetic sequence.
b. Graph the function and determine the domain.

**SOLUTION:**
a. The arithmetic sequence is 2, 4, 6,…. Find the common difference.

\[4 - 2 = 2\]

The sequence is increasing, so the common difference is positive: 2.

Write the equation for the \(n\)th term of an arithmetic sequence using the first term 2 and common difference 2.

\[
f_n = f_1 + (n - 1)d \\
= 2 + (n - 1)2 \\
= 2 + 2n - 2 \\
= 2n
\]

So, the function for this arithmetic sequence is \(f(n) = 2n\).

b. The points to graph are represented by \((n, f_n)\). So, the points are (1, 2); (2, 4); (3, 6).

The domain of the function is the number of hours spent at the park. So, the domain is \(\{1, 2, 3, 4, \ldots\}\).
3-5 Arithmetic Sequences as Linear Functions

23. CCSS MODELING The table shows how Ryan is paid for cutting 10-foot long 2x4 planks at his lumber yard job.

<table>
<thead>
<tr>
<th>Number of 10-ft 2x4 Planks Cut</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount Paid in Commission ($)</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
<td>40</td>
<td>48</td>
<td>56</td>
</tr>
</tbody>
</table>

a. Write a function to represent Ryan’s commission.
b. Graph the function and determine the domain.

**SOLUTION:**
a. The arithmetic sequence is 8, 16, 24, 32, 40, 48, 56, .... Find the common difference.
16 – 8 = 8
The sequence is increasing, so the common difference is positive: 8.
Write the equation for the nth term of an arithmetic sequence with first term 8 and common difference 8.

\[ f_n = f_1 + (n - 1)d \]
\[ = 8 + (n - 1)8 \]
\[ = 8 + 8n - 8 \]
\[ = 8n \]

So, the function for this arithmetic sequence is \( f(n) = 8n \).

b. The points to graph are represented by \((n, f_n)\). So, the points are (1, 8); (2, 16); (3, 24); (4, 32); (5, 40); (6, 48); (7, 56).

![Graph](image)

The domain of the function is the number of 10-foot 2x4 planks cut. So, the domain is \( \{1, 2, 3, 4, ...\} \).
3-5 Arithmetic Sequences as Linear Functions

24. The graph is a representation of an arithmetic sequence.

![Graph of an arithmetic sequence](image)

a. List the first five terms.
b. Write the formula for the \( n \)th term.
c. Write the function.

**SOLUTION:**
a. The points on the graph are \((1, -3); (2, -1); (3, 1); (4, 3); (5, 5)\). Because the terms are the second coordinate in each coordinate pair, the terms of the arithmetic sequence are \(-3, -1, 1, 3, 5\).

b. Find the common difference.
\[-1 - (-3) = 2\]

Write the equation for the \( n \)th term of an arithmetic sequence using the first term \(-3\) and common difference 2.

\[
a_n = a_1 + (n - 1)d
\]
\[
= -3 + (n - 1)2
\]
\[
= -3 + 2n - 2
\]
\[
= 2n - 5
\]

So, the formula for the \( n \)th term of the arithmetic sequence is \( a_n = 2n - 5 \).

c. \( f(n) = (n - 1)d + a_1 = 2n - 5 \)
3-5 Arithmetic Sequences as Linear Functions

25. **NEWSPAPERS** A local newspaper charges by the number of words for advertising.

![Newspaper advertisement](image)

Write a function to represent the advertising costs.

**SOLUTION:**
Find the common difference between sequential terms. Because the terms given skip by fives, divide the difference by five.

\[
8.75 - 7.50 = 1.25 \\
1.25 ÷ 5 = 0.25
\]

Subtract 0.25 nine times from the tenth term to find the first term.

\[
7.50 - 9(0.25) = 5.25
\]

Use the equation for the \(n\)th term of an arithmetic sequence to write a function using first term 5.25 and common difference 0.25.

\[
f(n) = 5.25 + (n - 1)0.25 \\
= 5.25 + 0.25n - 0.25 \\
= 0.25n + 5
\]

So, the function that represents the newspaper charges for advertising is \(f(n) = 0.25n + 5\).

26. The fourth term of an arithmetic sequence is 8. If the common difference is 2, what is the first term?

**SOLUTION:**
To find the first term, subtract 2 three times from the fourth term. \(8 - 2 - 2 - 2 = 2\). Or, subtract \((2 • 3)\) from the fourth term. \(8 - 6 = 2\). So the first term in the sequence is 2.

27. The common difference of an arithmetic sequence is \(-5\). If \(a_{12}\) is 22, what is \(a_1\)?

**SOLUTION:**
To find the first term, subtract \(-5\) eleven times from the twelfth term. \(22 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 77\). Or, subtract \((11 • -5)\) from the twelfth term. \(22 - (-55) = 77\). So the first term is 77.
28. The first four terms of an arithmetic sequence are 28, 20, 12, and 4. Which term of the sequence is −36?

**SOLUTION:**
Find the common difference.

\[28 - 20 = 8\]

The sequence is decreasing, so the common difference is negative: −8.

Write the equation for the \(n\)th term of an arithmetic sequence using the first term 28 and common difference −8.

\[a_n = a_1 + (n - 1)d\]

\[= 28 + (n - 1)(-8)\]

\[= 28 - 8n + 8\]

\[= 36 - 8n\]

Then, substitute −36 for \(a_n\) and evaluate for \(n\).

\[-36 = 36 - 8n\]

\[-36 + 36 = 36 + 36 - 8n\]

\[0 = 72 - 8n\]

\[0 + 8n = 72 - 8n + 8n\]

\[8n = 72\]

\[\frac{8n}{8} = \frac{72}{8}\]

\[n = 9\]

So, it is the 9\(^{th}\) term of the sequence that is −36.

29. **CARS** Jamal’s odometer of his car reads 24,521. If Jamal drives 45 miles every day, what will the odometer reading be after 25 days?

**SOLUTION:**
To find the 25th term, add 45 to \(a_0\) 25 times, or add \((45 \cdot 25)\) to \(a_0\).

\[24,521 + 1125 = 25,646\]

So after 25 days, the odometer reading will be 25,646 miles.
3-5 Arithmetic Sequences as Linear Functions

30. YEARBOOKS  The yearbook staff is unpacking a box of school yearbooks. The arithmetic sequence 281, 270, 259, 248 … represents the total number of ounces that the box weighs as each yearbook is taken out of the box.
   a. Write a function to represent this sequence.
   b. Determine the weight of each yearbook.
   c. If the box weighs at least 17 ounces empty and 292 ounces when it is full, how many yearbooks were in the box?

**SOLUTION:**

   a. The arithmetic sequence is 281, 270, 259, 248,…. Find the common difference.

   \[ 270 - 281 = -11 \]

   Write the equation for the \( n \)-th term of an arithmetic sequence using the first term 281 and common difference -11.

   \[
   f_n = a_1 + (n - 1)d \\
   = 281 + (n - 1) - 11 \\
   = 281 - 11n + 11 \\
   = -11n + 292
   \]

   So, the function for this arithmetic sequence is \( f(n) = -11n + 292 \).

   b. The weight decreases by 11 oz every time a book is taken out, so the common difference represents the weight of each yearbook, or 11 oz.

   c. To determine the number of yearbooks in a full box, subtract the weight of the empty box from the weight of the full box.

   \[ 292 - 17 = 275 \]

   Now, divide by the weight of each yearbook.

   \[ 275 ÷ 11 = 25 \]

   So there are 25 books in a full box.
3-5 Arithmetic Sequences as Linear Functions

31. **SPORTS** To train for an upcoming marathon, Olivia plans to run 3 miles per day for the first week and then increase the daily distance by a half mile each of the following weeks.

a. Write an equation to represent the nth term of the sequence.

b. If the pattern continues, during which week will she run 10 miles per day?

c. Is it reasonable to think that this pattern will continue indefinitely? Explain.

**SOLUTION:**

a. Write out the first few terms of the sequence.
3, 3.5, 4, 4.5, 5, 5.5, 6

The common difference is 0.5.

\[ a_n = a_1 + (n-1)d \quad \text{Formula for the nth term} \]

\[ a_n = 3 + (n-1)0.5 \quad \text{Replace } a_1 \text{ with 3 and } d \text{ with 0.5.} \]

\[ a_n = 3 + 0.5n - 0.5 \quad \text{Distributive Property} \]

\[ a_n = 2.5 + 0.5n \quad \text{Simplify.} \]

b. Solve for \( a_n = 10 \). Week 15

\[ a_n = 2.5 + 0.5n \quad \text{Formula for the nth term} \]

\[ 10 = 2.5 + 0.5n \quad \text{Replace } a_n \text{ with 10.} \]

\[ 10 - 2.5 = 2.5 - 2.5 + 0.5n \quad \text{Subtract 2.5 from each side.} \]

\[ 7.5 = 0.5n \quad \text{Simplify.} \]

\[ \frac{7.5}{0.5} = 0.5n \quad \text{Divide each side by 0.5.} \]

\[ 15 = n \quad \text{Simplify.} \]

In the 15th week will she run 10 miles per day.

c. She cannot continue with the daily increase of 0.5 miles each week. Eventually the number of miles ran per day will become unrealistic.

32. **OPEN ENDED**

Create an arithmetic sequence with a common difference of \(-10\).

**SOLUTION:**

Students’ answers may vary. Sample answer: Starting with a first term of 2, subtract 10 to get the next term. Then subtract 10 from the resulting number to get the next term, and so on.

\[ 2 - 10 = -8 \]

\[ -8 - 10 = -18 \]

\[ -18 - 10 = -28 \]

So, an arithmetic sequence with a common difference of \(-10\) is \(2, -8, -18, -28, \ldots\).
3.5 Arithmetic Sequences as Linear Functions

33. CCSS PERSEVERANCE
Find the value of \( x \) that makes \( x + 8, 4x + 6 \), and \( 3x \) the first three terms of an arithmetic sequence.

**SOLUTION:**
Find the common difference. The difference between the second and first terms should be equal to the difference between the third and second terms.

\[
(4x + 6) - (x + 8) = 3x - (4x + 6)
\]
\[
4x + 6 - x - 8 = 3x - 4x - 6
\]
\[
3x - 2 = -x - 6
\]
\[
3x - 2 + x = -x - 6 + x
\]
\[
4x - 2 = -6
\]
\[
4x - 2 + 2 = -6 + 2
\]
\[
4x = -4
\]
\[
4 \cdot \frac{x}{4} = - \frac{4}{4}
\]
\[
x = -1
\]

Substitute \(-1\) for \( x \).

\[
x + 8 = -1 + 8 = 7
\]
\[
4x + 6 = -4 + 6 = 2
\]
\[
3x = -3
\]

Check to see if the terms have a common difference.

\[
2 - 7 = -5
\]
\[
-3 - 2 = -5
\]

The terms have a common difference of \(-5\), so \( x = -1 \) makes these terms an arithmetic sequence.

34. REASONING
Compare and contrast the domain and range of the linear functions described by
\( Ax + By = C \) and \( a_n = a_1 + (n - 1)d \).

**SOLUTION:**
Sample answer: the domain of the function described by \( Ax + By = C \) is the set of all real numbers, and the range is either the set of all real numbers or a set of just one number when the graph is a horizontal line. For an arithmetic sequence, the domain is the set of all counting numbers. The range will be an infinite discrete set of real numbers if \( d \neq 0 \). If \( d = 0 \), then the range will be \( \{a_1\} \).
3-5 Arithmetic Sequences as Linear Functions

35. **CHALLENGE**
Determine whether each sequence is an arithmetic sequence. Write *yes* or *no*. Explain. If yes, find the common difference and the next three terms.

a. \(2x + 1, 3x + 1, 4x + 1\ldots\)

b. \(2x, 4x, 8x, \ldots\)

**SOLUTION:**
a. Find the common difference.
\[
(3x + 1) - (2x + 1) = 3x + 1 - 2x - 1
\]
\[
= x
\]
\[
(4x + 1) - (3x + 1) = 4x + 1 - 3x - 1
\]
\[
= x
\]
The common difference between terms is \(x\). So, to find the next term, add \(x\) to the last term. To find the next term, add \(x\) to the resulting number, and so on.

\[
4x + 1 + x = 5x + 1
\]
\[
5x + 1 + x = 6x + 1
\]
\[
6x + 1 + x = 7x + 1
\]
So, the next three terms are \(5x + 1, 6x + 1,\) and \(7x + 1\).

b. Try to find the common difference.
\[
4x - 2x = 2x
\]
\[
8x - 4x = 4x
\]
The difference between terms is not equal, so there is no common difference and the sequence is not an arithmetic sequence.
3-5 Arithmetic Sequences as Linear Functions

36. WRITING IN MATH How are graphs of arithmetic sequences and linear functions similar? different?

SOLUTION:
They are similar in that the graph of the terms of an arithmetic sequence lies on a line. Therefore, an arithmetic sequence can be represented by a linear function. They are different in that the domain of an arithmetic sequence is the set of natural numbers, while the domain of a linear function is all real numbers. Thus, arithmetic sequences are discrete, while linear functions are continuous.

Consider the graph of the arithmetic sequence \( a_n = 4n - 16 \).

For each \( n \), there is a point on the graph.

The graph of the linear function \( y = 4x - 16 \) is given below.
3-5 Arithmetic Sequences as Linear Functions

37. **GRIDDED RESPONSE**
   The population of Westerville is about 35,000. Each year the population increases by about 400. This can be represented by the following equation, where \( n \) represents the number of years from now and \( p \) represents the population.
   \[ p = 35,000 + 400n \]
   In how many years will the Westerville population be about 38,200?
   **SOLUTION:**
   Substitute the new population for \( p \) in the equation and evaluate for \( n \).
   \[
   \begin{align*}
   p &= 35,000 + 400n \\
   38,200 &= 35,000 + 400n \\
   38,200 - 35,000 &= 35,000 - 35,000 + 400n \\
   3200 &= 400n \\
   \frac{3200}{400} &= \frac{400n}{400} \\
   8 &= n
   \end{align*}
   \]
   So, the population will be about 38,200 in 8 years.

38. Which relation is a function?
   A \{(-5, 6), (4, -3), (2, -1), (4, 2)\}
   B \{(3, -1), (3, -5), (3, 4), (3, 6)\}
   C \{(-2, 3), (0, 3), (-2, -1), (-1, 2)\}
   D \{(-5, 6), (4, -3), (2, -1), (0, 2)\}
   **SOLUTION:**
   To be a function, then for each member in the domain, there is only one member of the range. You can also graph the points, and use the vertical line test.
   In Choice A, the points (4,3) and (4,2) have the same \( x \)-values but different \( y \)-values. Thus the relation in Choice A is not a function.
   In Choice B, all four points have 3 as the \( x \)-values but different \( y \)-values. Thus the relation in Choice B is not a function.
   In Choice C, (-2, 3) and (-2,1) have the same \( x \)-values but different \( y \)-values. Thus the relation in Choice C is not a function.
   The relation in choice D is a function because none of the \( x \) values are repeated. So, the correct choice is D.
3-5 Arithmetic Sequences as Linear Functions

39. Find the formula for the \(n\)th term of the arithmetic sequence.
   \(-7, -4, -1, 2, \ldots\)
   \(F \quad a_n = 3n - 4\)
   \(G \quad a_n = -7n + 10\)
   \(H \quad a_n = 3n - 10\)
   \(J \quad a_n = -7n + 4\)

   **SOLUTION:**
   Find the common difference.
   \(-4 - (-7) = 3\)
   Write the equation for the \(n\)th term of an arithmetic sequence using first term \(-7\) and common difference 3.
   \(a_n = a_1 + (n - 1)d\)
   \(= -7 + (n - 1)3\)
   \(= -7 + 3n - 3\)
   \(= 3n - 10\)
   So, the correct choice is \(H\).

40. **STATISTICS** A class received the following scores on the ACT. What is the difference between the median and the mode in the scores?
   \(18, 26, 20, 30, 25, 21, 32, 19, 22, 29, 29, 27, 24\)
   \(A \quad 1\)
   \(B \quad 2\)
   \(C \quad 3\)
   \(D \quad 4\)

   **SOLUTION:**
   Find the median by arranging the scores in sequential order and finding the value in the middle. The median is 25.
   Find the mode by looking for the value that occurs most frequently in the data. The mode is 29. Subtract to find the difference between the median and the mode. \(29 - 25 = 4\). So, the correct choice is \(D\).
41. **SOLUTION:**
Find the constant of variation using the point (1, 3).

\[ y = kx \]
\[ 3 = k(1) \]
\[ 3 = k \]

The constant of variation is 3. Find the slope of the line through the points (0, 0) and (1, 3).

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ = \frac{3 - 0}{1 - 0} \]
\[ = \frac{3}{1} \]
\[ = 3 \]

The slope of the line is 3.
3-5 Arithmetic Sequences as Linear Functions

SOLUTION:
Find the constant of variation using the point (−3, 4).

\[
y = kx \\
4 = k(−3) \\
\frac{4}{−3} = k \\
\frac{4}{3} = k
\]

The constant of variation is \(-\frac{4}{3}\).

Find the slope of the line through the points (0, 0) and (−3, 4).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \\
= \frac{4 - 0}{−3 - 0} \\
= \frac{4}{−3} \\
= −\frac{4}{3}
\]

The slope of the line is \(-\frac{4}{3}\).

Find the slope of the line that passes through each pair of points.

43. (5, 3), (−2, 6)

SOLUTION:

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \\
= \frac{6 - 3}{−2 - 5} \\
= \frac{3}{−7} \\
= −\frac{3}{7}
\]

The slope of the line is \(-\frac{3}{7}\).
3-5 Arithmetic Sequences as Linear Functions

44. (9, 2), (−3, −1)

**SOLUTION:**

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ = \frac{-1 - 2}{-3 - 9} \]

\[ = \frac{-3}{-12} \]

\[ = \frac{1}{4} \]

The slope of the line is \( \frac{1}{4} \).

45. (2, 8), (−2, −4)

**SOLUTION:**

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ = \frac{-4 - 8}{-2 - 2} \]

\[ = \frac{-12}{-4} \]

\[ = 3 \]

The slope of the line is 3.

**Solve each equation. Check your solution.**

46. 5x + 7 = −8

**SOLUTION:**

Solve.

\[ 5x + 7 = -8 \]

\[ 5x + 7 - 7 = -8 - 7 \]

\[ 5x = -15 \]

\[ \frac{5x}{5} = \frac{-15}{5} \]

\[ x = -3 \]

Check.

\[ 5x + 7 = -8 \]

\[ 5(-3) + 7 = -8 \]

\[ -15 + 7 = -8 \]

\[ -8 = -8 \]
3-5 Arithmetic Sequences as Linear Functions

47. \(8 = 2 + 3n\)

**SOLUTION:**

Solve.

\[
\begin{align*}
8 &= 2 + 3n \\
8 - 2 &= 2 - 2 + 3n \\
6 &= 3n \\
\frac{6}{3} &= \frac{3n}{3} \\
2 &= n
\end{align*}
\]

Check.

\[
\begin{align*}
8 &= 2 + 3n \\
8 &= 2 + 3(2) \\
8 &= 2 + 6 \\
8 &= 8
\end{align*}
\]

48. \(12 = \frac{c - 6}{2}\)

**SOLUTION:**

Solve.

\[
\begin{align*}
12 &= \frac{c - 6}{2} \\
12 \cdot 2 &= 2 \left( \frac{c - 6}{2} \right) \\
24 &= c - 6 \\
24 + 6 &= c - 6 + 6 \\
30 &= c
\end{align*}
\]

Check.

\[
\begin{align*}
12 &= \frac{c - 6}{2} \\
12 &= \frac{30 - 6}{2} \\
12 &= \frac{24}{2} \\
12 &= 12
\end{align*}
\]
3-5 Arithmetic Sequences as Linear Functions

49. **SPORTS** The most popular sports for high school girls are basketball and softball. Write and use an equation to find how many more girls play on basketball teams than on softball teams.

![Basketball and Softball](image)

**Basketball**
453,000 girls

**Softball**
369,000 girls

**SOLUTION:**
To write the equation, let \( d \) represent the difference between the number of girls on basketball teams and the number of girls on softball teams.

\[
453,000 - d = 369,000
\]

\[
453,000 - d + d = 369,000 + d
\]

\[
453,000 = 369,000 + d
\]

\[
453,000 - 369,000 = 369,000 - 369,000 + d
\]

\[
84,000 = d
\]

So, about 84,000 more girls play on basketball teams than on softball teams.

**Graph each point on the same coordinate plane.**

50. **A(2, 5)**

**SOLUTION:**

![Graph](image)
3-5 Arithmetic Sequences as Linear Functions

51. $B(-2, 1)$

**SOLUTION:**

52. $C(-3, -1)$

**SOLUTION:**

53. $D(0, 4)$

**SOLUTION:**
3-5 Arithmetic Sequences as Linear Functions

54. \( F(5, -3) \)

**SOLUTION:**

![Graph](image)

55. \( G(-5, 0) \)

**SOLUTION:**

![Graph](image)