2-8 Literal Equations and Dimensional Analysis

Solve each equation or formula for the variable indicated.

1. \(5a + c = -8a\), for \(a\)

   SOLUTION:
   
   \[
   \begin{align*}
   5a + c &= -8a & \text{Original} \\
   5a - 5a + c &= -8a - 5a & \text{Subtract} \ 5a \\
   c &= -13a & \text{Simplify} \\
   \frac{c}{-13} &= \frac{-13a}{-13} & \text{Divide by} -13 \\
   -\frac{c}{13} &= a & \text{Simplify}
   \end{align*}
   \]

2. \(7h + f = 2h + g\), for \(g\)

   SOLUTION:
   
   \[
   \begin{align*}
   7h + f &= 2h + g & \text{Original} \\
   7h - 2h + f &= 2h - 2h + g & \text{Subtract} \ 2h \\
   5h + f &= g & \text{Simplify}
   \end{align*}
   \]

3. \(\frac{k + m}{-7} = n\), for \(k\)

   SOLUTION:
   
   \[
   \begin{align*}
   \frac{k + m}{-7} &= n & \text{Original equation} \\
   -7 \left(\frac{k + m}{-7}\right) &= (-7)(n) & \text{Multiply by} -7 \\
   k + m &= -7n & \text{Simplify} \\
   k + m - m &= -7n - m & \text{Subtract} \ m \\
   k &= -7n - m & \text{Simplify}
   \end{align*}
   \]

4. \(q = p(r + s)\), for \(p\)

   SOLUTION:
   
   \[
   \begin{align*}
   q &= p(r + s) & \text{Original equation} \\
   \frac{q}{r + s} &= \frac{p(r + s)}{r + s} & \text{Divide each side by} \ r + s \\
   \frac{q}{r + s} &= p & \text{Simplify}
   \end{align*}
   \]
5. **PACKAGING** A soap company wants to use a cylindrical container to hold their new liquid soap.

![Diagram of a cylinder with formula: V = πr^2h]

**a.** Solve the formula for \( h \).

**b.** What is the height of a container if the volume is 56.52 cubic inches and the radius is 1.5 inches? Round to the nearest tenth.

**SOLUTION:**

**a.**

\[
V = \pi r^2 h \quad \text{Original equation}
\]

\[
\frac{V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2} \quad \text{Divide each side by} \pi r^2.
\]

\[
\frac{V}{\pi r^2} = h \quad \text{Simplify.}
\]

**b.** Replace \( V \) with 56.52 and \( r \) with 1.5 in the equation \( h = \frac{V}{\pi r^2} \). Then, solve for \( h \).

\[
h = \frac{V}{\pi r^2}
\]

\[
= \frac{56.52}{\pi (1.5)^2}
\]

\[
\approx 8.0
\]

So, the height is about 8 inches.

6. **SHOPPING** Scott found a rare video game on an online auction site priced at 35 Australian dollars. If the exchange rate is \$1 \text{ U.S.} = \$1.24 \text{ Australian}, find the cost of the game in United States dollars. Round to the nearest cent.

**SOLUTION:**

Since the given conversion relates U.S. dollars to Australian dollars, multiply 35 Australian dollars by the conversion factor such that the unit Australia dollars is divided out.

\[
35 \text{ Australian} \times \frac{\$1 \text{ U.S.}}{\$1.24 \text{ Australian}} = 35 \text{ U.S.}
\]

\[
= \frac{35 \text{ U.S.}}{1.24}
\]

\[
\approx 28.23 \text{ U.S.}
\]

The game costs $28.23 in United States dollars.
7. **CCSS PRECISION** A fisheye lens has a minimum focus range of 13.5 centimeters. If 1 centimeter is equal in length to about 0.39 inch, what is the minimum focus range of the lens in feet?

**SOLUTION:**
Since the given conversion relates centimeters to inches, first convert 13.5 centimeters to inches by multiplying by the conversion factor such that the unit centimeters is divided out.

\[
13.5 \text{ cm} \times \frac{0.39 \text{ in.}}{1 \text{ cm}} = 13.5(0.39 \text{ in.}) = 5.265 \text{ in.}
\]

There are 12 inches in 1 foot. Multiply by the conversion factor such that the unit inches is divided out.

\[
5.265 \text{ in.} \times \frac{1 \text{ ft}}{12 \text{ in.}} = \frac{5.265 \text{ ft}}{12} = 0.43875
\]

The minimum focus range of the lens is about 0.43875 foot.

**Solve each equation or formula for the variable indicated.**

8. \( u = vw + z \), for \( v \)

**SOLUTION:**

Original equation.

\[
u = vw + z
\]

Subtract \( z \) from each side.

\[
u - z = vw + z - z
\]

Simplify.

\[
u - z = vw
\]

Divide each side by \( w \).

\[
\frac{u - z}{w} = \frac{vw}{w}
\]

Simplify.

\[
\frac{u - z}{w} = v
\]

9. \( x = b - cd \), for \( c \)

**SOLUTION:**

Original equation.

\[
x = b - cd
\]

Subtract \( b \) from each side.

\[
x - b = b - b - cd
\]

Simplify.

\[
x - b = -cd
\]

Divide each side by \( -d \).

\[
\frac{x - b}{-d} = \frac{-cd}{-d}
\]

Simplify.

\[
\frac{x - b}{-d} = c
\]
2-8 Literal Equations and Dimensional Analysis

10. \( fg - 9h = 10j \), for \( g \)

**SOLUTION:**

\[
\begin{align*}
fg - 9h &= 10j \quad \text{Original equation} \\
fg - 9h + 9h &= 10j + 9h \quad \text{Add} \ h \ \text{to each side.} \\
fg &= 10j + 9h \quad \text{Simplify.} \\
\frac{fg}{f} &= \frac{10j + 9h}{f} \quad \text{Divide each side by} \ f. \\
g &= \frac{10j + 9h}{f} \quad \text{Simplify.}
\end{align*}
\]

11. \( 10m - p = -n \), for \( m \)

**SOLUTION:**

\[
\begin{align*}
10m - p &= -n \quad \text{Original} \\
10m - p + p &= -n + p \quad \text{Add} \ p. \\
10m &= -n + p \quad \text{Simplify.} \\
\frac{10m}{10} &= \frac{-n + p}{10} \quad \text{Divide by} \ 10. \\
m &= \frac{-n + p}{10} \quad \text{Simplify.}
\end{align*}
\]

12. \( r = \frac{2}{3}t + v \), for \( t \)

**SOLUTION:**

\[
\begin{align*}
r &= \frac{2}{3}t + v \quad \text{Original} \\
r - v &= \frac{2}{3}t + v - v \quad \text{Subtract} \ v. \\
r - v &= \frac{2}{3}t \quad \text{Simplify} \\
\left(\frac{3}{2}\right)(r - v) &= \left(\frac{3}{2}\right)(\frac{2}{3}t) \quad \text{Multiply by} \ \frac{3}{2} \\
\frac{3}{2}(r - v) &= t \quad \text{Simplify}
\end{align*}
\]
13. $\frac{5}{9}v + w = z$, for $v$

**SOLUTION:**

\[
\frac{5}{9}v + w = z \quad \text{Original}
\]

\[
\frac{5}{9}v + w - w = z - w \quad \text{Subtract } w.
\]

\[
\frac{5}{9}v = z - w \quad \text{Simplify}
\]

\[
\left(\frac{9}{5}\right)\left(\frac{5}{9}v\right) = \left(\frac{9}{5}\right)(z - w) \quad \text{Multiply by } \frac{9}{5}.
\]

\[
v = \frac{9}{5}(z - w) \quad \text{Simplify}
\]

14. \[
\frac{10ac - x}{11} = -3, \text{ for } a
\]

**SOLUTION:**

\[
\frac{10ac - x}{11} = -3 \quad \text{Original}
\]

\[
11\left(\frac{10ac - x}{11}\right) = 11(-3) \quad \text{Multiply by } 11.
\]

\[
10ac - x = -33 \quad \text{Simplify.}
\]

\[
10ac - x + x = -33 + x \quad \text{Add } x.
\]

\[
10ac = -33 + x \quad \text{Simplify.}
\]

\[
\frac{10ac}{10c} = \frac{-33 + x}{10c} \quad \text{Divide by } 10c.
\]

\[
a = \frac{-33 + x}{10c} \quad \text{Simplify.}
\]

15. \[
\frac{df + 10}{6} = g, \text{ for } f
\]

**SOLUTION:**

\[
\frac{df + 10}{6} = g \quad \text{Original}
\]

\[
6\left(\frac{df + 10}{6}\right) = 6(g) \quad \text{Multiply by } 6
\]

\[
 df + 10 = 6g \quad \text{Simplify.}
\]

\[
 df + 10 - 10 = 6g - 10 \quad \text{Subtract } 10.
\]

\[
 df = 6g - 10 \quad \text{Simplify.}
\]

\[
\frac{df}{d} = \frac{6g - 10}{d} \quad \text{Divide by } d.
\]

\[
f = \frac{6g - 10}{d} \quad \text{Simplify.}
\]
16. **FITNESS** The formula to compute a person’s body mass index is \( B = 703 \cdot \frac{w}{h^2} \). \( B \) represents the body mass index, \( w \) is the person’s weight in pounds, and \( h \) represents the person’s height in inches.

a. Solve the formula for \( w \).

\[
B = 703 \cdot \frac{w}{h^2}
\]

\[
B(h^2) = \left(703 \cdot \frac{w}{h^2}\right)(h^2)
\]

\[
Bh^2 = 703 \cdot w
\]

\[
\frac{Bh^2}{703} = \frac{703 \cdot w}{703}
\]

\[
\frac{Bh^2}{703} = w
\]

b. To find the weight to the nearest pound of a person who is 64 inches tall and has a body mass index of 21.45, replace \( h \) with 64 and \( B \) with 21.45 in the equation \( w = \frac{Bh^2}{703} \). Then, solve for \( w \).

\[
w = \frac{Bh^2}{703}
\]

\[
= \frac{21.45(64^2)}{703}
\]

\[
\approx 125
\]

A person who is 64 inches tall and has a body mass index of 21.45 is about 125 pounds.
2-8 Literal Equations and Dimensional Analysis

17. PHYSICS Acceleration is the measure of how fast a velocity is changing. The formula for acceleration is \( a = \frac{v_f - v_i}{t} \). \( a \) represents the acceleration rate, \( v_f \) is the final velocity, \( v_i \) is the initial velocity, and \( t \) represents the time in seconds.
   a. Solve the formula for \( v_i \).
   b. What is the final velocity of a runner who is accelerating at 2 feet per second squared for 3 seconds with an initial velocity of 4 feet per second?

   **SOLUTION:**
   a. 
   \[
   \begin{align*}
   a & = \frac{v_f - v_i}{t} \quad \text{Original} \\
   a(t) & = \frac{v_f - v_i}{t} \cdot t \quad \text{Multiply by \( t \)} \\
   at & = v_f - v_i \quad \text{Simplify.} \\
   at + v_i & = v_f - v_i + v_i \quad \text{Subtract \( v_i \).} \\
   at + v_i & = v_f \quad \text{Simplify.}
   \end{align*}
   \]

   b. To find the final velocity of a runner who is accelerating at 2 feet per second squared for 3 seconds with an initial velocity of 4 feet per second, replace \( a \) with 2, \( t \) with 3, and \( v_i \) with 4.

   \[
   v_f = at + v_i \\
   = 2(3) + 4 \\
   = 6 + 4 \\
   = 10
   \]

   The final velocity is 10 ft/s².

18. SWIMMING If each lap in a pool is 100 meters long, how many laps equal one mile? Round to the nearest tenth. (Hint: 1 foot ≈ 0.3048 meter)

   **SOLUTION:**
   There are 5280 feet in 1 mile. Multiply by the conversion factor such that the unit feet is divided out to find the number of meters in a mile.
   \[
   \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{0.3048 \text{ m}}{1 \text{ ft}} = \frac{1609.344 \text{ m}}{1 \text{ mi}}
   \]
   Because each lap is 100 meters long, multiply the number of meters in a mile by the conversion factor to find the number of laps in one mile.
   \[
   \frac{1609.344 \text{ m}}{1 \text{ mi}} \times \frac{1 \text{ lap}}{100 \text{ m}} = \frac{1609.344 \text{ laps}}{100 \text{ mi}}
   \]
   \[
   \approx \frac{16.1 \text{ laps}}{\text{mile}}
   \]
   About 16.1 laps equal 1 mile.
19. **CCSS PRECISION** How many liters of gasoline are needed to fill a 13.2-gallon tank? There are about 1.06 quarts per 1 liter. Round to the nearest tenth.

SOLUTION:
Since the given conversion relates quarts to liters, convert 13.2 gallons to quarts first by multiplying by the conversion factor so that the unit gallons is divided out.

\[
13.2 \text{ gallons} \times \frac{4 \text{ quarts}}{1 \text{ gallon}} = 52.8 \text{ quarts}
\]

Because there are 1.06 quarts per 1 liter, multiply 52.8 quarts by the conversion factor so that the unit quarts is divided out.

\[
52.8 \text{ quarts} \times \frac{1 \text{ liter}}{1.06 \text{ quarts}} = \frac{52.8 \text{ liters}}{1.06} \approx 49.8 \text{ liters}
\]

About 49.8 liters are needed to fill a 13.2-gallon tank.

Solve each equation or formula for the variable indicated.
20. \(-14n + q = rt - 4n\), for \(n\)

SOLUTION:

\[
\begin{align*}
-14n + q &= rt - 4n & \text{Original} \\
-14n + 4n + q &= rt - 4n + 4n & \text{Add 4n.} \\
-10n + q &= rt & \text{Simplify.} \\
-10n + q - q &= rt - q & \text{Subtract } q. \\
-10n &= rt - q & \text{Simplify.} \\
\frac{-10n}{-10} &= \frac{rt - q}{-10} & \text{Divide by } -10. \\
\frac{-10n}{-10} &= \frac{rt - q}{-10} & \text{Simplify.}
\end{align*}
\]

21. \(18t + 11v = w - 13t\), for \(t\)

SOLUTION:

\[
\begin{align*}
18t + 11v &= w - 13t & \text{Original} \\
18t + 13t + 11v &= w - 13t + 13t & \text{Add } 13t. \\
31t + 11v &= w & \text{Simplify.} \\
31t + 11v - 11v &= w - 11v & \text{Subtract } 11v. \\
31t &= w - 11v & \text{Simplify.} \\
\frac{31t}{31} &= \frac{w - 11v}{31} & \text{Divide by } 31. \\
t &= \frac{w - 11v}{31} & \text{Simplify.}
\end{align*}
\]
2-8 Literal Equations and Dimensional Analysis

22. \( ax + z = aw - y \), for \( a \)

**SOLUTION:**

\[
\begin{align*}
ax + z &= aw - y & \text{Original equation} \\
ax - aw + z &= aw - aw - y & \text{Subtract} aw \\
ax - aw + z &= -y & \text{Simplify.} \\
ax - aw + z - z &= -y - z & \text{Subtract} z \\
ax - aw &= -y - z & \text{Simplify.} \\
\frac{a(x-w)}{x-w} &= \frac{-y - z}{x-w} & \text{Distribute.} \\
&= \frac{-y - z}{x-w} & \text{Divide by} (x - w). \\
a &= \frac{-y - z}{x-w} & \text{Simplify.}
\end{align*}
\]

23. \( 10c - f = -13 + cd \), for \( c \)

**SOLUTION:**

\[
\begin{align*}
10c - f &= -13 + cd & \text{Original} \\
10c - f - cd &= -13 + cd - cd & \text{Subtract} cd \\
10c - cd - f &= -13 & \text{Simplify.} \\
10c - cd - f + f &= -13 + f & \text{Add} f \\
10c - cd &= -13 + f & \text{Simplify.} \\
c(10 - d) &= -13 + f & \text{Distribute.} \\
\frac{c(10 - d)}{10 - d} &= \frac{-13 + f}{10 - d} & \text{Divide.} \\
c &= \frac{-13 + f}{10 - d} & \text{Simplify.}
\end{align*}
\]
2-8 Literal Equations and Dimensional Analysis

Select an appropriate unit from the choices below and convert the rate to that unit.

| ft/s | mph | mm/s | km/s |

24. a car traveling at 36 ft/s

**SOLUTION:**
An appropriate unit is miles per hour.
Convert feet per second to miles per second by multiplying by the conversion unit so that the unit feet is divided out.

\[
\frac{36 \text{ ft}}{1 \text{ s}} \times \frac{1 \text{ mi}}{5280 \text{ ft}} = \frac{36 \text{ mi}}{5280 \text{ s}} \\
\approx \frac{0.0068 \text{ mi}}{\text{s}}
\]

Now, convert miles per second to miles per hour by multiplying by the conversion unit so that the unit seconds is divided out.

\[
\frac{0.0068 \text{ mi}}{1 \text{ s}} \times \frac{3600 \text{ s}}{1 \text{ hr}} \approx \frac{24.5 \text{ mi}}{1 \text{ hr}}
\]

The car travels at about 24.5 miles per hour.

25. a snail moving at 3.6 m/h

**SOLUTION:**
An appropriate unit is millimeters per second.
Convert meters per hour to millimeters per hour by multiplying by the conversion unit so that the unit meters is divided out.

\[
\frac{3.6 \text{ m}}{1 \text{ hr}} \times \frac{1000 \text{ mm}}{1 \text{ m}} = \frac{3600 \text{ mm}}{1 \text{ hr}}
\]

Now, convert millimeters per hour to millimeters per second by multiplying by the conversion factor so that the unit hours is divided out.

\[
\frac{3600 \text{ mm}}{1 \text{ hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}} = \frac{3600 \text{ mm}}{3600 \text{ s}} \\
= \frac{1 \text{ mm}}{1 \text{ s}}
\]

The snail moves at about 1 millimeter per second.
2-8 Literal Equations and Dimensional Analysis

26. a person walking at 3.4 mph

**SOLUTION:**
An appropriate unit is feet per second.
Convert miles per hour to feet per hour by multiplying by the conversion factor so that the unit miles is divided out.
\[
\frac{3.4 \text{ mph}}{1 \text{ hr}} \times \frac{5280 \text{ ft}}{1 \text{ mph}} = \frac{17,952 \text{ ft}}{1 \text{ hr}}
\]

Now, convert feet per hour to feet per second by multiplying by the conversion factor so that the unit hours is divided out.
\[
\frac{17,952 \text{ ft}}{1 \text{ hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}} = \frac{17,952 \text{ ft}}{3600 \text{ s}} \approx \frac{5.0 \text{ ft}}{1 \text{ s}}
\]

The person walks at about 5 feet per second.

27. a satellite moving at 234,000 m/min

**SOLUTION:**
An appropriate unit is kilometers per second.
Convert meters per minute to kilometers per minute by multiplying by the conversion factor so that the unit meters is divided out.
\[
\frac{234,000 \text{ m}}{1 \text{ min}} \times \frac{1 \text{ km}}{1000 \text{ m}} = \frac{234,000 \text{ km}}{1000 \text{ min}} = \frac{234 \text{ km}}{1 \text{ min}}
\]

Now, convert kilometers per minute to kilometers per second by multiplying by the conversion factor so that the unit minutes is divided out.
\[
\frac{234 \text{ km}}{1 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = \frac{234 \text{ km}}{60 \text{ s}} = \frac{3.9 \text{ km}}{1 \text{ s}}
\]
The satellite moves at 3.9 kilometers per second.
2-8 Literal Equations and Dimensional Analysis

28. **DANCING** The formula \( P = \frac{1.2W}{H^2} \) represents the amount of pressure exerted on the floor by a ballroom dancer’s heel. In this formula, \( P \) is the pressure in pounds per square inch, \( W \) is the weight of a person wearing the shoe in pounds, and \( H \) is the width of the heel of the shoe in inches.
   a. Solve the formula for \( W \).
   b. Find the weight of the dancer if the heel is 3 inches wide and the pressure exerted is 30 pounds per square inch.

**SOLUTION:**

a. \[
P = \frac{1.2W}{H^2} \quad \text{Original equation}
\]

\[
\left( H^2 \right) \cdot P = \left( H^2 \right) \cdot \left( \frac{1.2W}{H^2} \right) \quad \text{Multiply each side by } H^2
\]

\[
H^2 P = 1.2W \quad \text{Simplify.}
\]

\[
\frac{H^2 P}{1.2} = W \quad \text{Divide each side by 1.2.}
\]

b. To find the weight of the dancer if the heel is 3 inches wide and the pressure exerted is 30 pounds per square inch, replace \( H \) with 3 and \( P \) with 30 in the equation \( W = \frac{H^2 P}{1.2} \).

\[
W = \frac{3^2 \cdot 30}{1.2}
\]

\[
= \frac{270}{1.2}
\]

\[
= 225
\]

So, the dancer weighs 225 pounds.

**Write an equation and solve for the variable indicated.**

29. Seven less than a number \( t \) equals another number \( r \) plus 6. Solve for \( t \).

**SOLUTION:**

Seven less than a number \( t \) equals another number \( r \) plus 6.

\[
t - 7 = r + 6
\]

\[
t - 7 = r + 6 \quad \text{Original equation}
\]

\[
t - 7 + 7 = r + 6 + 7 \quad \text{Add 7 to each side}
\]

\[
t = r + 13 \quad \text{Simplify}
\]
2-8 Literal Equations and Dimensional Analysis

30. Ten plus eight times a number \( a \) equals eleven times another number \( d \) minus six. Solve for \( a \).

**SOLUTION:**

\[
\begin{align*}
10 + 8a &= 11d - 6 & \text{Original} \\
10 - 10 + 8a &= 11d - 6 - 10 & \text{Subtract 10.} \\
8a &= 11d - 16 & \text{Simplify.} \\
\frac{8a}{8} &= \frac{11d - 16}{8} & \text{Divide by 8.} \\
a &= \frac{11d - 16}{8} & \text{Simplify.}
\end{align*}
\]

31. Nine tenths of a number \( g \) is the same as seven plus two thirds of another number \( k \). Solve for \( k \).

**SOLUTION:**

\[
\begin{align*}
\frac{9}{10}g &= 7 + \frac{2}{3}k & \text{Original} \\
\frac{9}{10}g - 7 &= 7 - 7 + \frac{2}{3}k & \text{Subtract 7.} \\
\frac{9}{10}g - 7 &= \frac{2}{3}k & \text{Simplify.} \\
\frac{3}{2}\left[\frac{9}{10}g - 7\right] &= \frac{3}{2}\left(\frac{2}{3}k\right) & \text{Multiply by \( \frac{3}{2} \).} \\
\frac{3}{2}\left[\frac{9}{10}g - 7\right] &= k & \text{Simplify.}
\end{align*}
\]
2-8 Literal Equations and Dimensional Analysis

32. Three fourths of a number $p$ less two is five sixths of another number $r$ plus five. Solve for $r$.

**SOLUTION:**

Three fourths of a number $p$ less two is five sixths of another number $r$ plus five.

\[
\frac{3}{4} p - 2 = \frac{5}{6} r + 5
\]

\[
\frac{3}{4} p - 2 = \frac{5}{6} r + 5 \quad \text{original}
\]

\[
\frac{3}{4} p - 2 - 5 = \frac{5}{6} r + 5 - 5 \quad \text{subtract 5.}
\]

\[
\frac{3}{4} p - 7 = \frac{5}{6} r \quad \text{simplify.}
\]

\[
\frac{6}{5} \left[ \frac{3}{4} p - 7 \right] = \frac{6}{5} \left( \frac{5}{6} r \right) \quad \text{multiply by} \frac{6}{5}
\]

\[
\frac{6}{5} \left[ \frac{3}{4} p - 7 \right] = r \quad \text{simplify.}
\]

33. **GIFTS** Ashley has 214 square inches of paper to wrap a gift box. The surface area $S$ of the box can be found by using the formula $S = 2w(l + h) + 2lh$, where $w$ is the width of the box, $l$ is the length of the box, and $h$ is the height. If the length of the box is 7 inches and the width is 6 inches, how tall can Ashley’s box be?

**SOLUTION:**

To find how tall Ashley’s box can be, replace $S$ with 214, $1$ with $7$ and $w$ with 6 in the equation $S = 2w(l + h) + 2lh$. Then, solve for $h$.

\[
S = 2w(l + h) + 2lh \quad \text{surface area formula}
\]

\[
214 = 2(6)(7 + h) + 2(7)(h) \quad \text{substitute}
\]

\[
214 = 12(7 + h) + 14h \quad \text{simplify.}
\]

\[
214 = 84 + 12h + 14h \quad \text{distributive property}
\]

\[
130 = 12h + 14h \quad \text{subtract 84}
\]

\[
130 = 26h \quad \text{simplify}
\]

\[
\frac{130}{26} = \frac{26h}{26} \quad \text{divide by 26.}
\]

\[
5 = h \quad \text{simplify}
\]

So, the box can be 5 inches tall.
2-8 Literal Equations and Dimensional Analysis

34. **DRIVING** A car is driven $x$ miles a year and averages $m$ miles per gallon.
   a. Write a formula for $g$, the number of gallons used in a year.
   b. If the average price of gas is $p$ dollars per gallon, write a formula for the total gas cost $c$ in dollars for driving this car each year.
   c. Car A averages 15 miles per gallon on the highway, while Car B averages 35 miles per gallon on the highway. If you average 15,000 miles each year, how much money would you save on gas per week by using Car B instead of Car A if the cost of gas averages $3 per gallon? Explain.

**SOLUTION:**

a. The number of gallons $g$ multiplied by the number of miles per gallon $m$ equals the total number of miles $x$, so $gm = x$. Solving the formula for $g$ we get $g = \frac{x}{m}$.

b. If $g$ is the number of gallons and $p$ is the price per gallon, then the total price $c$ is equal to $p \times g$.

\[c = pg\]
\[c = pg\quad g = \frac{x}{m}\]

C. Find the cost for each car.

**Car A:**

\[c = \frac{px}{m}\quad \text{Formula for total cost.}\]
\[c = \frac{15000 \times 3}{15}\quad p = 15,000, x = 3, \text{ and } m = 15\]
\[c = 3000\quad \text{Simplify.}\]

**Car B:**

\[c = \frac{px}{m}\quad \text{Formula for total cost.}\]
\[c = \frac{15000 \times 3}{35}\quad p = 15,000, x = 3, \text{ and } m = 35\]
\[c = 1285.71\quad \text{Simplify.}\]

You would save about $3000 - 1286$ or $1714$ per year which is equivalent to about $\frac{1714}{52}$ or about $33$ per week.
2-8 Literal Equations and Dimensional Analysis

35. **CHALLENGE** The circumference of an NCAA women’s basketball is 29 inches, and the rubber coating is \( \frac{3}{16} \) inch thick. Use the formula \( v = \frac{4}{3} \pi r^3 \) where \( v \) represents the volume and \( r \) is the radius of the inside of the ball, to determine the volume of the air inside the ball. Round to the nearest whole number.

**SOLUTION:**

To find the volume of the air inside, first find the radius of the interior of the ball. The circumference of the ball is 29 inches. Replace \( C \) with 29 in the equation for circumference, \( C = 2\pi r \). Solve for \( r \).

\[
C = 2\pi r \\
29 = 2(\pi)r \\
\frac{29}{2\pi} = r \\
r \approx 4.61549
\]

The radius of the ball is about 4.61549 inches. To find the radius of the interior of the ball, subtract \( \frac{3}{16} \) from 4.61549, because the rubber coating is \( \frac{3}{16} \) inch thick. The radius of the interior of the ball is \( 4.61549 - \frac{3}{16} \) or about 4.42799 inches. To find the volume of the air inside, replace \( r \) with 3.86579 in the equation \( v = \frac{4}{3} \pi r^3 \) and solve for \( v \).

\[
v = \frac{4}{3} \pi r^3 \\
= \frac{4}{3} (\pi)(4.42799)^3 \\
\approx 364
\]

The volume of the air inside the ball is about 364 cubic inches.

36. **REASONING** Select an appropriate unit to describe the highway speed of a car and the speed of a caterpillar crawling on a tree. Can the same unit be used for both? Explain.

**SOLUTION:**

If you were to measure the average speed of a caterpillar, you would likely use a stop watch and a ruler to measure how many inches the caterpillar moves after a given number of seconds. This speed would be best measured in inches per second.

If you were to measure the average speed of a car on a highway, you would use the odometer to determine the number of miles that were driven in a given amount of time. Time could be measured in minutes or hours, but hours are most commonly used. The units used in this case would then be miles per hour.

The rate of a car is faster than a caterpillar. The rate of a car is best described by miles per hour and the rate of the caterpillar is best described by inches per second. So, you cannot use the same unit for each situation.
37. **ERROR ANALYSIS** Sandrea and Fernando are solving \(4a - 5b = 7\) for \(b\). Is either of them correct? Explain.

![SOLUTION](image)

**SOLUTION:**

Compare their solutions line by line. In the first line both Sandrea and Fernando have the problem written, but in the second line we already see a difference in their solution. Sandrea has \(-5b = 7 - 4a\) while Fernando has \(5b = 7 - 4a\). Looking back at the original equation \(4a - 5b = 7\) we see that there is a \(-5b\), and that Fernando has made a mistake.

Following the rest of their solutions, we see that no further mistakes are made, however the one error that Fernando made in the second line gives him the wrong solution.

Sandrea is correct because she performed each step correctly. Fernando omitted the negative sign from \(-5b\).

38. **OPEN ENDED** Write a formula for \(A\), the area of a geometric figure such as a triangle or rectangle. Then solve the formula for a variable other than \(A\).

**SOLUTION:**

Sample answer for a triangle: The area of a triangle is given by the formula \(A = \frac{1}{2}bh\). Solve it for \(b\).

\[
\begin{align*}
A &= \frac{1}{2}bh & \text{Area formula} \\
A(2) &= \left(\frac{1}{2}bh\right)(2) & \text{Multiply each side by 2} \\
2A &= bh & \text{Simplify} \\
\frac{2A}{h} &= \frac{bh}{h} & \text{Divide each side by} \ h \\
2\frac{A}{h} &= b & \text{Simplify}
\end{align*}
\]
39. CCSS PERSEVERANCE Solve each equation or formula for the variable indicated.
   a. \[ n = \frac{x+y-1}{xy} \] for \( x \)
   
   \[ n(xy) = \left( \frac{x+y-1}{xy} \right)(xy) \]
   Multiply by \( xy \).
   
   \[ nxy = x + y - 1 \]
   Simplify.
   
   \[ nxy - x = x - x + y - 1 \]
   Add \( x \).
   
   \[ xy^n - x = y - 1 \]
   Simplify.
   
   \[ x(y^n - 1) = y - 1 \]
   Distribute.
   
   \[ \frac{x(y^n - 1)}{y^n - 1} = \frac{y - 1}{y^n - 1} \]
   Divide by \( yn - 1 \).
   
   \[ x = \frac{y - 1}{y^n - 1} \]
   Simplify.

   b. \[ \frac{x+y}{x-y} = \frac{1}{2} \] for \( y \)
   
   \[ 2(x + y) = 1(x - y) \]
   Multiply by \( x - y \).
   
   \[ 2x + 2y = x - y \]
   Simplify.
   
   \[ 2x - 2x + 2y = x - 2x - y \]
   Subtract \( 2x \).
   
   \[ 2y = -x - y \]
   Simplify.
   
   \[ 2y + y = -x - y + y \]
   Add \( y \).
   
   \[ 3y = -x \]
   Simplify.
   
   \[ \frac{3y}{3} = \frac{-x}{3} \]
   Divide by \( 3 \).
   
   \[ y = -\frac{1}{3}x \]
   Simplify.

40. WRITING IN MATH Why is it helpful to be able to represent a literal equation in different ways?

   SOLUTION:
   You can rewrite the equation to isolate the unknown variable, substitute the given information, and then simplify to find the solution. For example, you will use the formula \( C = 2\pi r \) if you are given the radius and asked to find the circumference. However, if you are given the circumference, you would need to use the equation \( r = \frac{C}{2\pi} \) to find the radius.
2-8 Literal Equations and Dimensional Analysis

41. Eula is investing $6000, part at 4.5% interest and the rest at 6% interest. If \( d \) represents the amount invested at 4.5%, which expression represents the amount of interest earned in one year by the amount paying 6%?

A \( 0.06d \)  
B \( 0.06(d - 6000) \)  
C \( 0.06(d + 6000) \)  
D \( 0.06(6000 - d) \)

**SOLUTION:**

To find the amount of interest, multiply the amount invested at 6% by the interest rate in decimal form. So, the amount of interest is \( 0.06(6000 - d) \). Choice D is the correct answer.

42. Todd drove from Boston to Cleveland, a distance of 616 miles. His breaks, gasoline, and food stops took 2 hours. If his trip took 16 hours altogether, what was Todd’s average speed?

F 38.5 mph  
G 40 mph  
H 44 mph  
J 47.5 mph

**SOLUTION:**

To find Todd’s average speed, divide the number of miles he drove, 616, by the amount of time he spent driving, 16 – 2 or 14 hours.  
\[ \frac{616}{14} = 44 \]
Todd’s average speed was 44 miles per hour. So, Choice H is the correct answer.

43. **SHORT RESPONSE** Brian has 3 more books than Erika. Jasmine has triple the number of books that Brian has. Altogether Brian, Erika, and Jasmine have 22 books. How many books does Jasmine have?

**SOLUTION:**

Let \( b \) = the number of books Erika has. Let \( b + 3 \) = the number of books Brian has. Let \( 3(b + 3) \) = the number of books Jasmine has. So, \( b + b + 3 + 3(b + 3) = 22 \).

\[
\begin{align*}
b + b + 3 + 3(b + 3) &= 22 \\
b + b + 3 + 3b + 9 &= 22 \\
5b + 12 &= 22 \\
5b &= 10 \\
b &= 2
\end{align*}
\]

Jasmine has 3(2 + 3) or 15 books.
2-8 Literal Equations and Dimensional Analysis

44. **GEOMETRY** Which of the following best describes a plane?
   A a location having neither size nor shape
   B a flat surface made up of points having no depth
   C made up of points and has no thickness or width
   D a boundless, three–dimensional set of all points

   **SOLUTION:**
   A plane is a flat surface made up of points having no depth. Choice A defines a plane as a location having neither size nor shape. However, planes are not locations. Choice C describes a plane as made up of points and has no thickness or width. However, the plane is only made up of points that are on the plane. There can be points not on the specific plane. Choice D describes a plane as a boundless, three–dimensional set of all points. However, planes are two-dimensional. So, Choice B is the correct answer.

**Find the final price of each item.**

45. lamp: $120.00
   discount: 20%
   tax: 6%

   **SOLUTION:**
   Find the discount.
   \(0.20 \times 120 = 24\)

   Subtract the discount from the original price.
   \(120 - 24 = 96\)

   Find the tax on the discounted price.
   \(0.06 \times 96 = 5.76\)

   Add the tax to the discounted amount to find the total cost.
   \(96 + 5.76 = 101.76\)

   So, the total cost of the lamp is $101.76.
2-8 Literal Equations and Dimensional Analysis

46. dress: $70.00
discount: 30%
tax: 7%

**SOLUTION:**
Find the discount.
\[0.30 \times 70 = 21\]

Subtract the discount from the original price.
\[70 - 21 = 49\]

Find the tax on the discounted price.
\[0.07 \times 49 = 3.43\]

Add the tax to the discounted amount to find the total cost.
\[49 + 3.43 = 52.43\]

So, the total cost of the dress is $52.43.

47. camera: $58.00
discount: 25%
tax: 6.5%

**SOLUTION:**
Find the discount.
\[0.25 \times 58 = 14.5\]

Subtract the discount from the original price.
\[58 - 14.5 = 43.5\]

Find the tax on the discounted price.
\[0.065 \times 43.5 = 2.8275\]

Rounded to the nearest cent, the tax is $2.83.

Add the tax to the discounted amount to find the total cost.
\[43.5 + 2.83 = 46.33\]

So, the total cost of the camera is $46.33.
2-8 Literal Equations and Dimensional Analysis

48. jacket: $82.00
discount: 15%
tax: 6%

**SOLUTION:**
Find the discount.
\[ 0.15 \times 82 = 12.3 \]

Subtract the discount from the original price.
\[ 82 - 12.3 = 69.7 \]

Find the tax on the discounted price.
\[ 0.06 \times 69.7 = 4.182 \]

Rounded to the nearest cent, the tax is $4.18.

Add the tax to the discounted amount to find the total cost.
\[ 69.7 + 4.18 = 73.88 \]

So, the total cost of the jacket is $73.88.

49. comforter: $67.00
discount: 20%
tax: 6.25%

**SOLUTION:**
Find the discount.
\[ 0.20 \times 67 = 13.4 \]

Subtract the discount from the original price.
\[ 67 - 13.4 = 53.6 \]

Find the tax on the discounted price.
\[ 0.0625 \times 53.6 = 3.35 \]

Add the tax to the discounted amount to find the total cost.
\[ 53.6 + 3.35 = 56.95 \]

So, the total cost of the comforter is $56.95.
50. lawn mower: $720.00
   discount: 35%
   tax: 7%

   **SOLUTION:**
   Find the discount.
   0.35 \times 720 = 252

   Subtract the discount from the original price.
   720 - 252 = 468

   Find the tax on the discounted price.
   0.07 \times 468 = 32.76

   Add the tax to the discounted amount to find the total cost.
   468 + 32.76 = 500.76

   So, the total cost of the lawn mower is $500.76.

51. \( \frac{3}{4.5} = \frac{x}{2.5} \)

   **SOLUTION:**
   \[ \frac{3}{4.5} = \frac{x}{2.5} \quad \text{Original equation} \]
   \[ 3(2.5) = 4.5(x) \quad \text{Cross multiply.} \]
   \[ 7.5 = 4.5x \quad \text{Multiply.} \]
   \[ \frac{7.5}{4.5} = \frac{4.5x}{4.5} \quad \text{Divide each side by 4.5.} \]
   \[ 1.67 \approx x \quad \text{Simplify.} \]

52. \( \frac{2}{0.36} = \frac{7}{p} \)

   **SOLUTION:**
   \[ \frac{2}{0.36} = \frac{7}{p} \quad \text{Original equation} \]
   \[ 2(p) = 0.36(7) \quad \text{Cross multiply.} \]
   \[ 2p = 2.52 \quad \text{Multiply} \]
   \[ \frac{2p}{2} = \frac{2.52}{2} \quad \text{Divide each side by 2.} \]
   \[ p = 1.26 \quad \text{Simplify.} \]
53. \[ \frac{m}{9} = \frac{2.8}{4.9} \]

**SOLUTION:**

\[
\frac{m}{9} = \frac{2.8}{4.9} \quad \text{Original equation}
\]
\[
m(4.9) = 9(2.8) \quad \text{Cross multiply.}
\]
\[
4.9m = 25.2 \quad \text{Multiply.}
\]
\[
\frac{4.9m}{4.9} = \frac{25.2}{4.9} \quad \text{Divide each side by 4.9}
\]
\[
m \approx 5.14 \quad \text{Simplify.}
\]

54. **JOBS** Laurie mows lawns to earn extra money. She can mow at most 30 lawns in one week. She profits $15 on each lawn she mows. Identify a reasonable domain and range for this situation and draw a graph.

**SOLUTION:**

The domain is whole numbers between 0 and 30, because Laurie can mow at most 30 lawns. The range is whole numbers that are multiples of 15 between 0 and 450, because Laurie makes $15 for each lawn she mows.

D: \{0, 30\}; R: \{0, 450\}
55. **ENTERTAINMENT** Each member of the pit orchestra is selling tickets for the school musical. The trombone section sold 50 floor tickets and 90 balcony tickets. Write and evaluate an expression to find how much money the trombone section collected.

### School Musical

<table>
<thead>
<tr>
<th>Tickets</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor</td>
<td>$7.50</td>
</tr>
<tr>
<td>Balcony</td>
<td>$5.00</td>
</tr>
</tbody>
</table>

**SOLUTION:**

To find how much money the trombone section collected, multiply the number of each type of ticket by the cost of that ticket. Then, find the sum of those products. The expression $50(7.50) + 90(5.00)$ can be used to find how much the trombone section collected.

\[
50(7.50) + 90(5.00) = 375 + 450 = 825
\]

So, the trombone section collected $825.

### Solve each equation.

56. $8k + 9 = 7k + 6$

**SOLUTION:**

\[
\begin{align*}
8k + 9 &= 7k + 6 & \text{Original} \\
8k - 7k + 9 &= 7k - 7k + 6 & \text{Subtract } 7k \\
k + 9 &= 6 & \text{Simplify} \\
k + 9 - 9 &= 6 - 9 & \text{Subtract } 9 \\
k &= -3 & \text{Simplify}
\end{align*}
\]

57. $3 - 4q = 10q + 10$

**SOLUTION:**

\[
\begin{align*}
3 - 4q &= 10q + 10 & \text{Original} \\
3 - 4q + 4q &= 10q + 4q + 10 & \text{Add } 4q \\
3 &= 14q + 10 & \text{Simplify} \\
3 - 10 &= 14q + 10 - 10 & \text{Subtract } 10 \\
-7 &= 14q & \text{Simplify} \\
-\frac{7}{14} &= \frac{14q}{14} & \text{Divide by } 14 \\
-0.5 &= q & \text{Simplify}
\end{align*}
\]
58. \(\frac{3}{4}n + 16 = 2 - \frac{1}{8}n\)

**SOLUTION:**

\[
\begin{align*}
\frac{3}{4}n + 16 &= 2 - \frac{1}{8}n \\
\frac{6}{8}n + 16 &= 2 - \frac{1}{8}n \\
\frac{6}{8}n + 16 &= 2 - \frac{1}{8}n + \frac{1}{8}n \\
\end{align*}
\]

Add \(\frac{1}{8}n\).

\[
\begin{align*}
\frac{7}{8}n + 16 &= 2 \\
\frac{7}{8}n &= -14 \\
\frac{\frac{7}{8}n}{\frac{7}{8}} &= \frac{-14}{\frac{7}{8}} \\
n &= -16
\end{align*}
\]

Simplify.

59. \(\frac{1}{4} - \frac{2}{3}y = \frac{3}{4} - \frac{1}{3}y\)

**SOLUTION:**

\[
\begin{align*}
\frac{1}{4} - \frac{2}{3}y &= \frac{3}{4} - \frac{1}{3}y \\
\frac{1}{4} - \frac{2}{3}y + \frac{2}{3}y &= \frac{3}{4} - \frac{1}{3}y + \frac{2}{3}y \\
\frac{1}{4} &= \frac{3}{4} + \frac{1}{3}y \\
\frac{1}{4} - \frac{3}{4} &= \frac{3}{4} - \frac{3}{4} + \frac{1}{3}y \\
-\frac{1}{2} &= \frac{1}{3}y \\
\frac{3}{\frac{1}{2}} &= \frac{3}{\frac{1}{3}y} \\
-1.5 &= y
\end{align*}
\]

Multiply by 3.
2-8 Literal Equations and Dimensional Analysis

60. \( 4(2a - 1) = -10(a - 5) \)

**SOLUTION:**

\[
\begin{align*}
4(2a - 1) &= -10(a - 5) & \text{Original} \\
8a - 4 &= -10a + 50 & \text{Distribute.} \\
8a + 10a - 4 &= -10a + 10a + 50 & \text{Add } 10a. \\
18a - 4 &= 50 & \text{Simplify.} \\
18a - 4 + 4 &= 50 + 4 & \text{Add 4.} \\
18a &= 54 & \text{Simplify.} \\
\frac{18a}{18} &= \frac{54}{18} & \text{Divide by } 18. \\
a &= 3 & \text{Simplify.}
\end{align*}
\]

61. \( 2(w - 3) + 5 = 3(w - 1) \)

**SOLUTION:**

\[
\begin{align*}
2(w - 3) + 5 &= 3(w - 1) & \text{Original} \\
2w - 6 + 5 &= 3w - 3 & \text{Distribute.} \\
2w - 1 &= 3w - 3 & \text{Simplify.} \\
2w - 2w - 1 &= 3w - 2w - 3 & \text{Subtract } 2w. \\
-1 &= w - 3 & \text{Simplify.} \\
-1 + 3 &= w - 3 + 3 & \text{Add 3.} \\
2 &= w & \text{Simplify.}
\end{align*}
\]