2-5 Solving Equations Involving Absolute Value

Evaluate each expression if \( f = 3, g = -4, \text{ and } h = 5. \)

1. \(|3 - h| + 13\)

**SOLUTION:**

\[
|3 - h| + 13 = |3 - 5| + 13 \quad \text{Replace } h \text{ with } 5.
\]

\[
= |-2| + 13 \quad 3 - 5 \text{ is } -2
\]

\[
= 2 + 13 \quad |1 - 2| \text{ is } 2.
\]

\[
= 15 \quad \text{Simplify.}
\]

2. \(16 - |g + 9|\)

**SOLUTION:**

\[
16 - |g + 9| = 16 - |4 + 9| \quad \text{Replace } g \text{ with } -4.
\]

\[
= 16 - |5| \quad -4 + 9 \text{ is } 5.
\]

\[
= 16 - 5 \quad |5| \text{ is } 5.
\]

\[
= 11 \quad \text{Simplify.}
\]

3. \(|f + g| - h\)

**SOLUTION:**

\[
|f + g| - h = |3 + (-4)| - 5 \quad f = 3, g = -4, h = 5
\]

\[
= |3 - 4| - 5 \quad \text{Rewrite.}
\]

\[
= |-1| - 5 \quad 3 - 4 \text{ is } -1.
\]

\[
= 1 - 5 \quad |-1| \text{ is } 1.
\]

\[
= -4 \quad \text{Simplify.}
\]

Solve each equation. Then graph the solution set.

4. \(|n + 7| = 5\)

**SOLUTION:**

Case 1:

\[
n + 7 = 5
\]

\[
n + 7 - 7 = 5 - 7 \quad \text{Subtract } 7 \text{ from each side.}
\]

\[
n = -2 \quad \text{Simplify.}
\]

Case 2:

\[
n + 7 = -5
\]

\[
n + 7 - 7 = -5 - 7 \quad \text{Subtract } 7 \text{ from each side.}
\]

\[
n = -12 \quad \text{Simplify.}
\]

The solution set is \{-2, -12\}. 

---

-13-12-11-10-9-8-7-6-5-4-3-2-1
2-5 Solving Equations Involving Absolute Value

5. \(|3z - 3| = 9\)

**SOLUTION:**

Case 1:

\[3z - 3 = 9\]

\[3z = 12\]  \text{Simplify.}

\[\frac{3z}{3} = \frac{12}{3}\]

\[z = 4\]  \text{Simplify.}

Case 2:

\[3z - 3 = -9\]

\[3z = -6\]  \text{Simplify.}

\[\frac{3z}{3} = \frac{-6}{3}\]

\[z = -2\]  \text{Simplify.}

The solution set is \(\{4, -2\}\).

6. \(|4n - 1| = -6\)

**SOLUTION:**

\(|4n - 1| = -6\) means the distance between 4n and 1 is -6. Since distance cannot be negative, the solution is the empty set \(\emptyset\).

7. \(|b + 4| = 2\)

**SOLUTION:**

Case 1:

\[b + 4 = 2\]

\[b + 4 - 4 = 2 - 4\]  \text{Subtract 4 from each side.}

\[b = -2\]  \text{Simplify.}

Case 2:

\[b + 4 = -2\]

\[b + 4 - 4 = -2 - 4\]  \text{Subtract 4 from each side}

\[b = -6\]  \text{Simplify.}

The solution set is \(\{-6, -2\}\).
8. \( |2t - 4| = 8 \)

**SOLUTION:**

Case 1:

\[ 2t - 4 = 8 \]

\[ 2t - 4 + 4 = 8 + 4 \quad \text{Add 4 to each side.} \]

\[ 2t = 12 \quad \text{Simplify.} \]

\[ \frac{2t}{2} = \frac{12}{2} \quad \text{Divide each side by 2.} \]

\[ t = 6 \quad \text{Simplify.} \]

Case 2:

\[ 2t - 4 = -8 \]

\[ 2t - 4 + 4 = -8 + 4 \quad \text{Add 4 to each side.} \]

\[ 2t = -4 \quad \text{Simplify.} \]

\[ \frac{2t}{2} = \frac{-4}{2} \quad \text{Divide each side by 2.} \]

\[ t = -2 \quad \text{Simplify.} \]

The solution set is \(-2, 6\).

9. \( |5h + 2| = -8 \)

**SOLUTION:**

\( |5h + 2| = -8 \) means the distance between \( 5h \) and \(-2 \) is \(-8 \). Since distance cannot be negative, the solution is the empty set \( \emptyset \).
2-5 Solving Equations Involving Absolute Value

10. **FINANCIAL LITERACY** For a company to invest in a product, they must believe they will receive a 12% return on investment (ROI) plus or minus 3%. Write an equation to find the least and the greatest ROI they believe they will receive.

**SOLUTION:**
Let \( x \) represent the ROI. The equation to find the least and greatest ROI is \( |x - 12| = 3 \).

Case 1:
\[
x - 12 = 3
\]
\[
x - 12 + 12 = 3 + 12 \quad \text{Add 12 to each side.}
\]
\[
x = 15 \quad \text{Simplify.}
\]

Case 2:
\[
x - 12 = -3
\]
\[
x - 12 + 12 = -3 + 12 \quad \text{Add 12 to each side}
\]
\[
x = 9 \quad \text{Simplify.}
\]

The solution set is \{9, 15\}. So, the least ROI is 9% and the greatest ROI is 15%.

**Write an equation involving absolute value for each graph.**

11.

**SOLUTION:**
Use the equation \( |x - n| = a \), where \( n \) is the midpoint and \( a \) the distance between the midpoint and each endpoint.

The midpoint between \(-2\) and \(4\) is \(1\). The distance between the midpoint and each endpoint is \(3\). So, the equation is \( |x - 1| = 3 \).

12.

**SOLUTION:**
Use the equation \( |x - n| = a \), where \( n \) is the midpoint and \( a \) the distance between the midpoint and each endpoint.

The midpoint between \(-9\) and \(3\) is \(-3\). The distance between the midpoint and each endpoint is \(6\). So, the equation is \( |x + 3| = 6 \).
2-5 Solving Equations Involving Absolute Value

Evaluate each expression if \( a = -2, \ b = -3, \ c = 2, \ x = 2.1, \ y = 3, \) and \( z = -4.2. \)

13. \(|2x + z| + 2y\

**SOLUTION:**
Replace \( x \) with \(2.1, \ y \) with \(3, \) and \( z \) with \(-4.2.\)

\[
|2x + z| + 2y \\
= |2(2.1) + (-4.2)| + 2(3) & \text{Substitute.} \\
= |4.2 - 4.2| + 6 & \text{Multiply.} \\
= |0| + 6 & \text{Subtract.} \\
= 0 + 6 & |0| \text{ is 0.} \\
= 6 & \text{Add}
\]

14. \(4a - |3b + 2c|\

**SOLUTION:**
Replace \( a \) with \(-2, \ b \) with \(-3, \) and \( c \) with \(2.\)

\[
4a - |3b + 2c| \\
= 4(-2) - |3(-3) + 2(2)| & \text{Substitute.} \\
= 4(-2) - | -9 + 4| & \text{Multiply.} \\
= -8 - | -9 + 4| & \text{Multiply.} \\
= -8 - |-5| & \text{Subtract.} \\
= -8 - 5 & | -5| \text{ is 5.} \\
= -13 & \text{Subtract.}
\]

15. \(-|5a + c| + |3y + 2z|\

**SOLUTION:**
Replace \( a \) with \(-2, \ c \) with \(2, \ y \) with \(3, \) and \( z \) with \(-4.2.\)

\[
-|5a + c| + |3y + 2z| \\
= -|5(-2) + (2)| + |3(3) + 2(-4.2)| & \text{Substitute.} \\
= -|-10 + 2| + |9 - 8.4| & \text{Multiply.} \\
= -|-8| + |0.6| & \text{Simplify.} \\
= -8 + 0.6 & |0.6| \text{ is 0.6} \\
= -7.4 & \text{Simplify.}
\]
2-5 Solving Equations Involving Absolute Value

16. \(-a + |2x - a|\)

**SOLUTION:**
Replace \(a\) with \(-2\) and \(x\) with \(2.1\).

\[-a + |2x - a|\]
\[= -(-2) + |2(2.1) - (-2)| \quad \text{Substitute.}\]
\[= 2 + |4.2 + 2| \quad \text{Multiply.}\]
\[= 2 + |6.2| \quad \text{Simplify.}\]
\[= 2 + 6.2 \quad |1 - 1| = 1.\]
\[= 8.2 \quad \text{Simplify.}\]

17. \(|y - 2z| - 3\)

**SOLUTION:**
Replace \(y\) with \(3\) and \(z\) with \(-4.2\).

\[|y - 2z| - 3\]
\[= |3 - 2(-4.2)| - 3 \quad \text{Substitute.}\]
\[= |3 + 8.4| - 3 \quad \text{Multiply.}\]
\[= |11.4| - 3 \quad \text{Simplify.}\]
\[= 11.4 - 3 \quad |11.4| = 11.4.\]
\[= 8.4 \quad \text{Simplify.}\]

18. \(3|3b - 8c| - 3\)

**SOLUTION:**
Replace \(b\) with \(-3\) and \(c\) with \(2\).

\[3|3b - 8c| - 3\]
\[= 3|3(-3) - 8(2)| - 3 \quad \text{Substitute.}\]
\[= 3|-9 - 16| - 3 \quad \text{Multiply.}\]
\[= 3|-25| - 3 \quad |-25| = 25.\]
\[= 3(25) - 3 \quad |-25| = 25.\]
\[= 75 - 3 \quad \text{Multiply.}\]
\[= 72 \quad \text{Simplify.}\]
2-5 Solving Equations Involving Absolute Value

19. \(|2x - z| + 6b\)

**SOLUTION:**
Replace \(x\) with 2.1, \(z\) with \(-4.2\), and \(b\) with \(-3\).

\[
|2x - z| + 6b = |2(2.1) - (4.2)| + 6(-3) \quad \text{Substitute.}
\]
\[
= |4.2 + 2.1| - 18 \quad \text{Multiply.}
\]
\[
= |6.4| - 18 \quad \text{Simplify.}
\]
\[
= 6.4 - 18 \quad |6.4| \text{ is 6.4.}
\]
\[
= -9.6 \quad \text{Simplify.}
\]

20. \(-3|z| + 2(a + y)\)

**SOLUTION:**
Replace \(z\) with \(-4.2\), \(a\) with \(-2\), and \(y\) with 3.

\[
-3|z| + 2(a + y) = -3|4.2| + 2(-2 + 3) \quad \text{Substitute.}
\]
\[
= -3|4.2| + 2(1) \quad \text{Simplify.}
\]
\[
= -3(4.2) + 2(1) \quad |4.2| \text{ is 4.2}
\]
\[
= -12.6 + 2 \quad \text{Multiply.}
\]
\[
= -10.6 \quad \text{Simplify.}
\]

21. \(-4|c - 3| + 2|z - a|\)

**SOLUTION:**
Replace \(c\) with 2, \(z\) with \(-4.2\), and \(a\) with \(-2\).

\[
-4|c - 3| + 2|z - a| = -4|2 - 3| + 2|4.2 - (-2)| \quad \text{Substitute.}
\]
\[
= -4|1| + 2|4.2 + 2| \quad \text{Simplify.}
\]
\[
= -4(1) + 2|4.4| \quad |1| \text{ is 1.}
\]
\[
= -4 + 2(4.4) \quad |4.4| \text{ is 4.4}
\]
\[
= 0.4 \quad \text{Multiply.}
\]
2-5 Solving Equations Involving Absolute Value

Solve each equation. Then graph the solution set.

22. \(|n - 3| = 5\)

\textbf{SOLUTION:}

Case 1:
\[n - 3 = 5\]
\[n - 3 + 3 = 5 + 3 \quad \text{Add 3 to each side.}\]
\[n = 8 \quad \text{Simplify.}\]

Case 2:
\[n - 3 = -5\]
\[n - 3 + 3 = -5 + 3 \quad \text{Add 3 to each side.}\]
\[n = -2 \quad \text{Simplify.}\]

The solution set is \(\{8, -2\}\).

23. \(|f + 10| = 1\)

\textbf{SOLUTION:}

Case 1:
\[f + 10 = 1\]
\[f + 10 - 10 = 1 - 10 \quad \text{Subtract 10 from each side.}\]
\[f = -9 \quad \text{Simplify.}\]

Case 2:
\[f + 10 = -1\]
\[f + 10 - 10 = -1 - 10 \quad \text{Subtract 10 from each side.}\]
\[f = -11 \quad \text{Simplify.}\]

The solution set is \(\{-11, -9\}\).

24. \(|v - 2| = -5\)

\textbf{SOLUTION:}\n
\(|v - 2| = -5\) means the distance between \(v\) and 2 is \(-5\). Since distance cannot be negative, the solution is the empty set \(\emptyset\).
2-5 Solving Equations Involving Absolute Value

25. \(|4t - 8| = 20\)

**SOLUTION:**

Case 1:
\[4t - 8 = 20\]
\[4t - 8 + 8 = 20 + 8 \quad \text{Add 8 to each side.}\]
\[4t = 28 \quad \text{Simplify.}\]
\[\frac{4t}{4} = \frac{28}{4} \quad \text{Divide each side by 4.}\]
\[t = 7 \quad \text{Simplify.}\]

Case 2:
\[4t - 8 = -20\]
\[4t - 8 + 8 = -20 + 8 \quad \text{Add 8 to each side.}\]
\[4t = -12 \quad \text{Simplify.}\]
\[\frac{4t}{4} = \frac{-12}{4} \quad \text{Divide each side by 4.}\]
\[t = -3 \quad \text{Simplify.}\]

The solution set is \(\{7, -3\}\).

26. \(|8w + 5| = 21\)

**SOLUTION:**

Case 1:
\[8w + 5 = 21\]
\[8w + 5 - 5 = 21 - 5 \quad \text{Subtract 5 from each side.}\]
\[8w = 16 \quad \text{Simplify.}\]
\[\frac{8w}{8} = \frac{16}{8} \quad \text{Divide each side by 8.}\]
\[w = 2 \quad \text{Simplify.}\]

Case 2:
\[8w + 5 = -21\]
\[8w + 5 - 5 = -21 - 5 \quad \text{Subtract 5 from each side.}\]
\[8w = -26 \quad \text{Simplify.}\]
\[\frac{8w}{8} = \frac{-26}{8} \quad \text{Divide each side by 8.}\]
\[w = -3.25 \quad \text{Simplify.}\]

The solution set is \(\{2, -3.25\}\).
2-5 Solving Equations Involving Absolute Value

27. \(|6y - 7| = -1\)

**SOLUTION:**

\(|6y - 7| = -1\) means the distance between 6y and 7 is -1. Since distance cannot be negative, the solution is the empty set \(\emptyset\).

28. \(\left| \frac{1}{2}x + 5 \right| = -3\)

**SOLUTION:**

\(\left| \frac{1}{2}x + 5 \right| = -3\) means the distance between \(\frac{1}{2}x\) and -5 is -3. Since distance cannot be negative, the solution is the empty set \(\emptyset\).

29. \(|-2y + 6| = 6\)

**SOLUTION:**

Case 1:

\(-2y + 6 = 6\)

\(-2y + 6 - 6 = 6 - 6\) Subtract 6 from each side.

\(-2y = 0\) Simplify.

\(\frac{-2y}{-2} = \frac{0}{-2}\) Divide each side by -2.

\(y = 0\) Simplify.

Case 2:

\(-2y + 6 = -6\)

\(-2y + 6 - 6 = -6 - 6\) Subtract 6 from each side.

\(-2y = -12\) Simplify.

\(\frac{-2y}{-2} = \frac{-12}{-2}\) Divide each side by -2.

\(y = 6\) Simplify.

The solution set is \(\{0, 6\}\).
2-5 Solving Equations Involving Absolute Value

30. $\left|\frac{3}{4}a - 3\right| = 9$

**SOLUTION:**

Case 1:

\[\frac{3}{4}a - 3 = 9\]

\[\frac{3}{4}a - 3 + 3 = 9 + 3\quad \text{Add 3 to each side}\]

\[\frac{3}{4}a = 12\quad \text{Simplify}\]

\[\frac{4}{3} \left(\frac{3}{4}a\right) = \frac{4}{3} (12)\quad \text{Multiply each side by } \frac{4}{3}\]

\[a = \frac{48}{3}\quad \text{Simplify}\]

\[a = 16\]

Case 2:

\[\frac{3}{4}a - 3 = -9\]

\[\frac{3}{4}a - 3 + 3 = -9 + 3\quad \text{Add 3 to each side}\]

\[\frac{3}{4}a = -6\quad \text{Simplify}\]

\[\frac{4}{3} \left(\frac{3}{4}a\right) = \frac{4}{3} (-6)\quad \text{Multiply each side by } \frac{4}{3}\]

\[a = -\frac{24}{3}\quad \text{Simplify}\]

\[a = -8\]

The solution set is \{16, -8\}.
2-5 Solving Equations Involving Absolute Value

31. **SURVEY** The circle graph at the right shows the results of a survey that asked, “How likely is it that you will be rich some day?” If the margin of error is ±4%, what is the range of the percent of teens who say it is very likely that they will be rich?

![Circle Graph]

**SOLUTION:**

First, write an equation. Let \( x \) represent the percent of teens who say it is very likely that they will be rich. The equation is \(|x - 15| = 4\).

**Case 1:**

\[
x - 15 = 4
\]

\[
x - 15 + 15 = 4 + 15
\]

\[
x = 19
\]

**Case 2:**

\[
x - 15 = -4
\]

\[
x - 15 + 15 = -4 + 15
\]

\[
x = 11
\]

Therefore, the range of percent of teens who say that it is very likely that they will be rich is 11% to 19%.
32. **CHEERLEADING** For competition, the cheerleading team is preparing a dance routine that must last 4 minutes, with a variation of ±5 seconds.
   a. Find the least and greatest possible times for the routine in minutes and seconds.
   b. Find the least and greatest possible times in seconds.

**SOLUTION:**
   a. First, write an equation for the length of the dance routine. Let \( x \) represent the time of the routine. Five seconds is \( \frac{5}{60} \) or \( \frac{1}{12} \) of one minute. So, the equation is \( |x - 4| = \frac{1}{12} \).

   Case 1:
   \[
   x - 4 = \frac{1}{12}
   \]
   \[
   x = 4 + \frac{1}{12} = 4\frac{1}{12} \text{ min}
   \]

   Case 2:
   \[
   x - 4 = -\frac{1}{12}
   \]
   \[
   x = 3\frac{11}{12} \text{ min}
   \]

   Therefore, the least possible time for the routine is 4 \( \frac{11}{12} \) minutes or 3 minutes 55 seconds. The greatest possible time is 4 \( \frac{1}{12} \) minutes or 4 minutes 5 seconds.

   b. Convert the time to seconds by multiplying by 60.
   \[
   4\frac{11}{12} \cdot 60 = 235
   \]
   \[
   4\frac{1}{12} \cdot 60 = 245
   \]
   The least possible time is 235 seconds and the greatest possible time is 245 seconds.

**Write an equation involving absolute value for each graph.**

33. **SOLUTION:**
   Use the equation \( |x - n| = a \), where \( n \) is the midpoint and \( a \) the distance between the midpoint and each endpoint. The midpoint between 4 and \(-4\) is 0. The distance between the midpoint and each point \( a \) is 4. So, the equation is \( |x| = 4 \).
2-5 Solving Equations Involving Absolute Value

34.

SOLUTION:
Use the equation \(|x - n| = a\), where \(n\) is the midpoint and \(a\) the distance between the midpoint and each endpoint. The midpoint between 6 and \(-6\) is 0. The distance between the midpoint and each point \(a\) is 6. So, the equation is \(|x| = 6\).

35.

SOLUTION:
Use the equation \(|x - n| = a\), where \(n\) is the midpoint and \(a\) the distance between the midpoint and each endpoint. The midpoint between 5 and \(-3\) is 1. The distance between the midpoint and each point \(a\) is 4. So, the equation is \(|x - 1| = 4\).

36.

SOLUTION:
Use the equation \(|x - n| = a\), where \(n\) is the midpoint and \(a\) the distance between the midpoint and each endpoint. The midpoint between 2 and \(-6\) is \(-2\). The distance between the midpoint and each point \(a\) is 4. So, the equation is \(|x + 2| = 4\).
2-5 Solving Equations Involving Absolute Value

Solve each equation. Then graph the solution set.

37. \( \left| -\frac{1}{2}b - 2 \right| = 10 \)

**SOLUTION:**

Case 1:

\[-\frac{1}{2}b - 2 = 10 \]

\[-\frac{1}{2}b - 2 + 2 = 10 + 2 \quad \text{Add 2 to each side.} \]

\[-\frac{1}{2}b = 12 \quad \text{Simplify.} \]

\[-2\left( -\frac{1}{2}b \right) = -2(12) \quad \text{Multiply each side by \(-2\).} \]

\[b = -24 \quad \text{Simplify.} \]

Case 2:

\[-\frac{1}{2}b - 2 = -10 \]

\[-\frac{1}{2}b - 2 + 2 = -10 + 2 \quad \text{Add 2 to each side} \]

\[-\frac{1}{2}b = -8 \quad \text{Simplify.} \]

\[-2\left( -\frac{1}{2}b \right) = -2(-8) \quad \text{Multiply each side by \(-2\).} \]

\[b = 16 \quad \text{Simplify.} \]

The solution set is \{-24, 16\}.
38. \(|-4d + 6| = 12\)

**SOLUTION:**

Case 1:

\(-4d + 6 = 12\)

\(-4d + 6 - 6 = 12 - 6\) Subtract 6 from each side.

\(-4d = 6\) Simplify.

\(\dfrac{-4d}{-4} = \dfrac{6}{-4}\) Divide each side by \(-4\).

\(d = -\dfrac{3}{2}\) Simplify.

Case 2:

\(-4d + 6 = -12\)

\(-4d + 6 - 6 = -12 - 6\) Subtract 6 from each side.

\(-4d = -18\) Simplify.

\(\dfrac{-4d}{-4} = \dfrac{-18}{-4}\) Divide each side by \(-4\)

\(d = \dfrac{9}{2}\) Simplify.

The solution set is \(\left\{\dfrac{3}{2}, \dfrac{9}{2}\right\}\).
39. \(|5f - 3| = 12\)

**SOLUTION:**

Case 1:

\[ 5f - 3 = 12 \]

\[ 5f - 3 + 3 = 12 + 3 \quad \text{Add 3 to each side.} \]

\[ 5f = 15 \quad \text{Simplify.} \]

\[ \frac{5f}{5} = \frac{15}{5} \quad \text{Divide each side by 5.} \]

\[ f = 3 \quad \text{Simplify.} \]

Case 2:

\[ 5f - 3 = -12 \]

\[ 5f - 3 + 3 = -12 + 3 \quad \text{Add 3 to each side.} \]

\[ 5f = -9 \quad \text{Simplify.} \]

\[ \frac{5f}{5} = \frac{-9}{5} \quad \text{Divide each side by 5.} \]

\[ f = \frac{-9}{5} \quad \text{Simplify.} \]

The solution set is \( \left\{ 3, \frac{-9}{5} \right\} \).

40. \( 2|h| - 3 = 8 \)

**SOLUTION:**

Isolate the absolute value first.

\[ 2|h| - 3 = 8 \]

\[ 2|h| - 3 + 3 = 8 + 3 \quad \text{Add 3 to each side.} \]

\[ 2|h| = 11 \quad \text{Simplify.} \]

\[ \frac{2|h|}{2} = \frac{11}{2} \quad \text{Divide each side by 2.} \]

\[ |h| = 5.5 \quad \text{Simplify.} \]

Case 1:

\[ h = 5.5 \]

Case 2:

\[ h = -5.5 \]

The solution set is \( \{5.5, -5.5\} \).
2-5 Solving Equations Involving Absolute Value

41. \(4 - 3|q| = 10\)

**SOLUTION:**
Isolate the absolute value first.

\[
4 - 3|q| = 10 \\
4 - 4 - 3|q| = 10 - 4 \quad \text{Subtract 4.} \\
-3|q| = 6 \quad \text{Simplify.} \\
\frac{-3|q|}{-3} = \frac{6}{-3} \quad \text{Divide by } -3 \quad \text{Simplify.} \\
|q| = -2 \quad \text{Simplify.}
\]

\(|q| = -2\) means the distance between \(q\) and 0 is -2. Since distance cannot be negative, there is no solution.

42. \(\frac{4}{|p|} + 12 = 14\)

**SOLUTION:**
Isolate the absolute value first.

\[
\frac{4}{|p|} + 12 = 14 \\
\frac{4}{|p|} + 12 - 12 = 14 - 12 \quad \text{Subtract 12.} \\
\frac{4}{|p|} = 2 \quad \text{Simplify.} \\
\frac{4}{|p|}(|p|) = 2(|p|) \quad \text{Multiply by } |p|. \\
4 = 2|p| \quad \text{Simplify.} \\
\frac{4}{2} = \frac{2|p|}{2} \quad \text{Divide by 2.} \\
2 = |p| \quad \text{Simplify.}
\]

Case 1:
\(p = 2\)

Case 2:
\(p = -2\)

The solution set is \(\{2, -2\}\).
2-5 Solving Equations Involving Absolute Value

43. **CCSS SENSE-MAKING** The 4×400 relay is a race where 4 runners take turns running 400 meters, or one lap around the track.

   **a.** If a runner runs the first leg in 52 seconds plus or minus 2 seconds, write an equation to find the fastest and slowest times.

   **b.** If the runners of the second and third legs run their laps in 53 seconds plus or minus 1 second a piece, write an equation to find their fastest and slowest times.

   **c.** Suppose the runner of the fourth leg is the fastest on the team. If he runs an average of 50.5 seconds plus or minus 1.5 seconds, what are the team’s fastest and slowest times?

   **SOLUTION:**

   **a.** Let \( x \) represent the time for the first runner to run the race. The equation is \( |x - 52| = 2 \).

   **Case 1:**
   
   \[
   x - 52 = 2 \\
   x = 54
   \]

   **Case 2:**
   
   \[
   x - 52 = -2 \\
   x = 50
   \]

   The first runner’s fastest time is 50 seconds, and the slowest time is 54 seconds.

   **b.** Let \( x \) represent the time for the second or third runner to run the race. The equation is \( |x - 53| = 1 \).

   **Case 1:**
   
   \[
   x - 53 = 1 \\
   x = 54
   \]

   **Case 2:**
   
   \[
   x - 53 = -1 \\
   x = 52
   \]

   The second or third runner’s fastest time is 52 seconds, and their slowest time is 54 seconds.

   **c.** Let \( x \) represent the time for the last runner to run the race. The equation is \( |x - 50.5| = 1.5 \).

   **Case 1:**
   
   \[
   x - 50.5 = 1.5 \\
   x = 52
   \]

   **Case 2:**
   
   \[
   x - 50.5 = -1.5 \\
   x = 49
   \]

   The fastest time for the team would be all four runners’ fastest times: \( 50 + 52 + 52 + 49 = 203 \) seconds.

   The slowest time for the team would be all four runners’ slowest times: \( 54 + 54 + 54 + 52 = 214 \) seconds.
2-5 Solving Equations Involving Absolute Value

44. **FASHION** To allow for a model’s height, a designer is willing to use models that require him to change hems either up or down 2 inches. The length of the skirts is 20 inches.
   a. Write an absolute value equation that represents the length of the skirts.
   b. What is the range of the lengths of the skirts?
   c. If a 20-inch skirt was fitted for a model that is 5 feet 9 inches tall, will the designer use a 6-foot-tall model?

**SOLUTION:**
   a. Let \( t \) represent the length of the skirt. The equation is \(|t - 20| = 2\).
   
   b. Case 1:
   \[
   |t - 20| = 2
   \]
   \[
   t - 20 = 2
   \]
   \[
   t - 20 + 20 = 2 + 20 \quad \text{Add 20 to each side.}
   \]
   \[
   t = 22 \quad \text{Simplify.}
   \]
   Case 2:
   \[
   t - 20 = -2
   \]
   \[
   t - 20 + 20 = -2 + 20 \quad \text{Add 20 to each side.}
   \]
   \[
   t = 18 \quad \text{Simplify.}
   \]
   The range of the lengths of the skirts is 18 inches to 22 inches.
   
   c. The difference in the models’ height is 3 inches, but the difference in the skirt hem length is 2 inches. Therefore, no, because the difference in heights of the models is greater than the difference in hem length of the skirt, the designer will not use this model.

45. **CCSS PRECISION** Speedometer accuracy can be affected by many details such as tire diameter and axle ratio. For example, there is variation of \( \pm 3 \) miles per hour when calibrated at 50 miles per hour.
   a. What is the range of actual speeds of the car if calibrated at 50 miles per hour?
   b. A speedometer calibrated at 45 miles per hour has an accepted variation of \( \pm 1 \) mile per hour. What can we conclude from this?

**SOLUTION:**
   a. Let \( x \) represent the speed of the car. The equation is \(|x - 50| = 3\).
   
   Case 1:
   \[
   x - 50 = 3
   \]
   \[
   x - 50 + 50 = 3 + 50 \quad \text{Add 50 to each side.}
   \]
   \[
   x = 53 \quad \text{Simplify.}
   \]
   Case 2:
   \[
   x - 50 = -3
   \]
   \[
   x - 50 + 50 = -3 + 50 \quad \text{Add 50 to each side.}
   \]
   \[
   x = 47 \quad \text{Simplify.}
   \]
   Therefore, the range of actual speeds of the car is 47 to 53 miles per hour.
   
   b. Sample answer: The speedometer was calibrated more accurately than the speedometer for part a. Many factors could affect the calibration of a speedometer and cause one speedometer to be more accurate than another.
2-5 Solving Equations Involving Absolute Value

Write an equation involving absolute value for each graph.

46. SOLUTION:

Use the equation $|x - n| = a$, where $n$ is the midpoint and $a$ the distance between the midpoint and each endpoint.
The midpoint between $-3\frac{3}{4}$ and $4\frac{1}{4}$ $n$ is $\frac{1}{4}$. The distance between the midpoint and each endpoint $a$ is 4. So, the equation is $|x - \frac{1}{4}| = 4$.

47. SOLUTION:

Use the equation $|x - n| = a$, where $n$ is the midpoint and $a$ the distance between the midpoint and each endpoint.
The midpoint between $-1\frac{1}{2}$ and $1\frac{1}{2}$ $n$ is 0. The distance between the midpoint and each endpoint $a$ is $\frac{1}{2}$. So, the equation is $|x| = 1\frac{1}{2}$.

48. SOLUTION:

Use the equation $|x - n| = a$, where $n$ is the midpoint and $a$ the distance between the midpoint and each endpoint.
The midpoint between $-2\frac{1}{2}$ and $3\frac{1}{2}$ $n$ is $\frac{1}{2}$. The distance between the midpoint and each endpoint $a$ is 3. So, the equation is $|x - \frac{1}{2}| = 3$.

49. SOLUTION:

Use the equation $|x - n| = a$, where $n$ is the midpoint and $a$ the distance between the midpoint and each endpoint.
The midpoint between 0 and $\frac{1}{4}$ $n$ is $\frac{1}{4}$. The distance between the midpoint and each endpoint $a$ is $\frac{1}{4}$. So, the equation is $|x - \frac{1}{4}| = \frac{1}{4}$.
2-5 Solving Equations Involving Absolute Value

50. **SOLUTION:**
Use the equation $|x - n| = a$, where $n$ is the midpoint and $a$ the distance between the midpoint and each endpoint. 

The midpoint between 0 and $1\frac{1}{3}$ is $\frac{2}{3}$. The distance between the midpoint and each endpoint $a$ is $\frac{2}{3}$. So, the equation is $|x - \frac{2}{3}| = \frac{2}{3}$.

51. **SOLUTION:**
Use the equation $|x - n| = a$, where $n$ is the midpoint and $a$ the distance between the midpoint and each endpoint. 

The midpoint between $-1\frac{1}{3}$ and $\frac{2}{3}$ is $\frac{1}{3}$. The distance between the midpoint and each endpoint $a$ is 1. So, the equation is $|x + \frac{1}{3}| = 1$.

52. **MUSIC** A CD will record an hour and a half of music plus minus 3 minutes for time to change tracks.

   a. Write an absolute value equation that represents the recording time.
   b. What is the range of time in minutes that the CD could run?
   c. Graph the possible times on a number line.

   **SOLUTION:**
   a. Let $t$ represent the time. The equation is $|t - 90| = 3$.

   b. Case 1:
   $$t - 90 = 3$$
   $$t - 90 + 90 = 3 + 90 \quad \text{Add 90 to each side.}$$
   $$t = 93 \quad \text{Simplify.}$$

   Case 2:
   $$t - 90 = -3$$
   $$t - 90 + 90 = -3 + 90 \quad \text{Add 90 to each side.}$$
   $$t = 87 \quad \text{Simplify.}$$

   The range of time is 87 to 93 minutes.

   c. 

   ![Graph showing the range of time on a number line]
53. ACOUSTICS The Red Rocks Amphitheater located in the Red Rock Park near Denver, Colorado, is the only naturally occurring amphitheater. The acoustic qualities here are such that a maximum of 20,000 people, plus or minus 1000, can hear natural voices clearly.
   a. Write an equation involving an absolute value that represents the number of people that can hear natural voices at Red Rocks Amphitheater.
   b. Find the maximum and minimum number of people that can hear natural voices clearly in the amphitheater.
   c. What is the range of people in part b?

SOLUTION:
   a. Let \( h \) = the number of people that can clearly hear voices. The equation is \( |h - 20,000| = 1000 \).

   b. Case 1:
      \[
      h - 20,000 = 1000
      \]
      \[
      h - 20,000 + 20,000 = 1000 + 20,000
      \]
      \[
      h = 21,000
      \]
   Case 2:
      \[
      h - 20,000 = -1000
      \]
      \[
      h - 20,000 + 20,000 = -1000 + 20,000
      \]
      \[
      h = 19,000
      \]
   Therefore, the maximum number of people that can clearly hear voices is 21,000 and the minimum number of people is 19,000.

   c. The range of people that can clearly hear natural voices is between 19,000 and 21,000.

54. BOOK CLUB The members of a book club agree to read within ten pages of the last page of the chapter. The chapter ends on page 203.
   a. Write an absolute value equation that represents the pages where club members could stop reading
   b. Write the range of the pages where the club members could stop reading.

SOLUTION:
   a. Let \( p \) represent the number of pages. The equation is \( |p - 203| = 10 \).

   b. Case 1:
      \[
      p - 203 = 10
      \]
      \[
      p - 203 + 203 = 10 + 203 \quad \text{Add 203 to each side.}
      \]
      \[
      p = 213 \quad \text{Simplify.}
      \]
   Case 2:
      \[
      p - 203 = -10
      \]
      \[
      p - 203 + 203 = -10 + 203 \quad \text{Add 203 to each side.}
      \]
      \[
      p = 193 \quad \text{Simplify.}
      \]
   So, the range of pages where the club members could stop reading is 193 to 213.
2-5 Solving Equations Involving Absolute Value

55. **SCHOOL** Teams from Washington and McKinley High Schools are competing in an academic challenge. A correct response on a question earns 10 points and an incorrect response loses 10 points. A team earns 0 points on an unattempted question. There are 5 questions in the math section.

   a. What are the maximum and minimum scores a team can earn on the math section?
   b. Suppose the McKinley team has 160 points at the start of the math section. Write and solve an equation that represents the maximum and minimum scores the team could have at the end of the math section.
   c. What are all of the possible scores that a school can earn on the math section?

**SOLUTION:**

   a. Each school could answer every question correctly, earning 50 points. They could also answer every question incorrectly, earning -50 points.

   b. Let \( m \) = the score on the math section. The maximum distance between \( m \) and the initial score of 160 is 50, so \( |m - 160| = 50 \).

   c. Scores of 50, 40, 30, 20, 10, 0, -10, -20, -30, -40 and -50 are possible. The ways to achieve each score as (correct, incorrect, unanswered) are:

   50: (5, 0, 0);
   40: (4, 0, 1);
   30: (3, 0, 2) or (4, 1, 0);
   20: (2, 0, 3) or (3, 1, 1);
   10: (1, 0, 4), (2, 1, 2), or (3, 2, 0);
   0: (0, 0, 5), (1, 1, 3), or (2, 2, 1);
   -10: (0, 1, 4), (1, 2, 2), or (2, 3, 0);
   -20: (0, 2, 3) or (1, 4, 0);
   -30: (0, 3, 2) or (1, 4, 0);
   -40: (0, 4, 1);
   -50: (0, 5, 0).

56. **OPEN ENDED** Describe a real-world situation that could be represented by the absolute value equation \( |x - 4| = 10 \).

**SOLUTION:**

Sample answer: Let \( x \) = the temperature at night. The answer to the equation should be the base temperature plus or minus the range, which is 4 degrees with a range of plus or minus 10 degrees. Therefore, the temperature at night is \( 4 \pm 10 \) degrees.
2-5 Solving Equations Involving Absolute Value

CCSS STRUCTURE Determine whether the following statements are sometimes, always, or never true, if \( c \) is an integer. Explain your reasoning

57. The value of \( |x+1| \) is greater than zero.

**SOLUTION:**
Set \( |x+1| \) equal to zero to determine if there is a value that makes the expression equal to zero.

\[
\begin{align*}
|x+1| &= 0 \\
x+1 &= 0 \\
x &= -1
\end{align*}
\]

The statement the value of \( |x+1| \) is greater than zero is sometimes true. If \( x = -1 \), then the value is equal to zero.

58. The solution of \( |x+c| = 0 \) is greater than 0.

**SOLUTION:**
The statement the solution of \( |x+c| = 0 \) is greater than 0 is sometimes true. For example, if \( c \) is negative, then \( x \) must be positive.

\[
\begin{align*}
|x+(-2)| &= 0 \\
x+(-2) &= 0 \\
x &= 2
\end{align*}
\]

However, if \( c \) is positive, then \( x \) is negative.

\[
\begin{align*}
|x+2| &= 0 \\
x+2 &= 0 \\
x &= -2
\end{align*}
\]

So, when \( c \) is a negative value, \( x \) is greater than zero.

59. The inequality \( |x|+c < 0 \) has no solution.

**SOLUTION:**

\[
\begin{align*}
|x|+c &< 0 \\
|x|+c-c &< -c \\
|x| &< -c
\end{align*}
\]

The statement the inequality \( |x|+c < 0 \) has no solution is sometimes true. When \( c \) is a negative value, the inequality is true.
2-5 Solving Equations Involving Absolute Value

60. The value of \(|x + c| + c\) is greater than zero.

**SOLUTION:**
The statement *the value of \(|x + c| + c\) is greater than zero* is sometimes true. For example, if \(x = 1\) and \(c = -5\), then:

\[
\begin{align*}
|x + c| + c &= |1 - 5| - 5 \\
&= |-4| - 5 \\
&= 4 - 5 \\
&= -1
\end{align*}
\]

This answer is not greater than zero. However, if \(x = 1\) and \(c = 2\), then:

\[
\begin{align*}
|x + c| + c &= |1 + 2| + 2 \\
&= |3| + 2 \\
&= 3 + 2 \\
&= 5
\end{align*}
\]

Therefore, the answer is sometimes true.

61. **REASONING** Explain why an absolute value can never be negative

**SOLUTION:**
An absolute value represents a distance from zero on a number line. A distance can never be a negative number. Therefore, an absolute value can never be a negative number.

62. **CHALLENGE** Use the sentence \(x = 7 \pm 4.6\).
   a. Describe the values of \(x\) that make the sentence true.
   b. Translate the sentence into an equation involving absolute value.

**SOLUTION:**
   a. \(x = 7 \pm 4.6\)
   There are two solutions to the equation.
   \[
   x = 7 + 4.6 \quad \text{and} \quad x = 7 - 4.6
   \]
   \[
   = 11.6 \quad \text{and} \quad = 2.4
   \]
   So, the values of \(x\) are 2.4, 11.6.

   b. The midpoint between 2.4 and 11.6 is 7. The distance between the midpoint and each endpoint is 4.6, so the equation is \(|x - 7| = 4.6\)
63. **ERROR ANALYSIS** Alex and Wesley are solving $|x + 5| = -3$. Is either of them correct? Explain your reasoning.

**SOLUTION:**
Wesley’s answer is correct. The absolute value of a number cannot be a negative number, so his answer is correct.

64. **WRITING IN MATH** Explain why there are either two, one, or no solutions for absolute value equations. Demonstrate an example of each possibility.

**SOLUTION:**
Sample answer: There are two solutions when the absolute value is equal to a positive number. There is one solution if the equation indicates that the absolute value is equal to zero. There are no solutions if the absolute value is equal to a negative number.

Absolute values are distances, which can never be negative numbers. Two solutions: $|x| = 10$, because $|10| = 10$ and $|-10| = 10$. One solution: $|x| = 0$, because $|0| = 0$. No solution: $|x| = -10$, because the distance a number $x$ is from 0 cannot be negative.

65. Which equation represents the second step of the solution process?
Step 1 $4(2x + 7) - 6 = 3x$
Step 2 ________
Step 3 $5x + 28 - 6 = 0$
Step 4 $5x = -22$
Step 5 $x = -4.4$

A $4(2x - 6) + 7 = 3x$
B $4(2x + 1) = 3x$
C $8x + 7 - 6 = 3x$
D $8x + 28 - 6 = 3x$

**SOLUTION:**
Answers A and B are not the correct answer because the part of the problem in the parentheses has changed. If you distribute the 4 to the parentheses you get $4(2x) + 4(7)$ or $8x + 28$. Answer C is not correct since 4 was not multiplied by the 7. Therefore, choice D is correct.
2-5 Solving Equations Involving Absolute Value

66. **GEOMETRY** The area of a circle is $25\pi$ square centimeters. What is the circumference?

- **F** $625\pi$ cm
- **G** $50\pi$ cm
- **H** $5\pi$ cm
- **J** $10\pi$ cm

**SOLUTION:**
The area of the circle is represented by $A = \pi r^2$ and the circumference can be represented by $C = 2\pi r$. First, use the area to find the radius of the circle.

\[
A = \pi r^2
\]
\[
25\pi = \pi r^2
\]
\[
\frac{25\pi}{\pi} = \frac{\pi r^2}{\pi}
\]
\[
25 = r^2
\]
\[
\sqrt{25} = r
\]
\[
5 = r
\]

Now, substitute 5 for $r$ to find the circumference.

\[
C = 2\pi r
\]
\[
= 2\pi(5)
\]
\[
= 10\pi
\]

The circumference of the circle is $10\pi$ cm. Choice J is correct.

67. Tanya makes $5 an hour and 15% commission of the total dollar value on cosmetics she sells. Suppose Tanya’s commission is increased to 17%. How much money will she make if she sells $300 worth of product and works 30 hours?

- **A** $201
- **B** $226
- **C** $255
- **D** $283

**SOLUTION:**
Let $t$ represent the amount of money Tanya can earn if her commission is increased.

\[
t = 5(30) + 0.17(300)
\]
\[
= 150 + 51
\]
\[
= 201
\]

Tanya will earn $201. Choice A is correct.
68. **EXTENDED RESPONSE** John’s mother has agreed to take him driving every day for two weeks. On the first day, John drives for 20 minutes. Each day after that, John drives 5 minutes more than the day before.

- **a.** Write an expression for the minutes John drives on the nth day. Explain.
- **b.** For how many minutes will John drive on the last day? Show your work.
- **c.** John’s driver’s education teacher requires that each student drive for 30 hours with an adult outside of class. Will John’s sessions with his mother fulfill this requirement?

**SOLUTION:**

- **a.** Each day, John will drive an additional 5 minutes. This can be represented by $5(n)$. The first day he drove 20 minutes. Solve the the equation $x + 5(1) = 20$ for $x$.

\[
x + 5(1) = 20
\]

\[
x + 5 = 20 \quad \text{Simplify.}
\]

\[
x + 5 - 5 = 20 - 5 \quad \text{Subtract 5 from each side.}
\]

\[
x = 15 \quad \text{Simplify.}
\]

Thus, the equation would be $15 + 5(n)$.

A formula for the nth term of the sequence is $15 + 5(n)$ because John will drive 20 minutes on the first day and then an additional 5 minutes for each day after that.

- **b.** Two weeks is equal to 14 days, so substitute 14 for $n$.

\[
15 + 5(n) = 15 + 5(14) \quad \text{Replace $n$ with 14.}
\]

\[
= 15 + 70 \quad \text{Multiply.}
\]

\[
= 85 \quad \text{Simplify.}
\]

John would drive for 85 minutes on the last day.

- **c.** On the last day, John has driven $20 + 25 + 30 + 35 + 40 + 45 + 50 + 55 + 60 + 65 + 70 + 75 + 80 + 85$ or 735 minutes. The class requires 30 hours, which is equal to 1800 minutes. So, John will not fulfill his requirement.
2-5 Solving Equations Involving Absolute Value

Write and solve an equation for each sentence.

69. One half of a number increased by 16 is four less than two thirds of the number.

**SOLUTION:**
\[
\frac{1}{2}n + 16 = \frac{2}{3}n - 4 \quad \text{Original}
\]
\[
\frac{1}{2}n + 16 + 4 = \frac{2}{3}n - 4 + 4 \quad \text{Add 4.}
\]
\[
\frac{1}{2}n + 20 = \frac{2}{3}n \quad \text{Simplify.}
\]
\[
\frac{3}{6}n + 20 = \frac{4}{6}n \quad \text{Use LCD.}
\]
\[
\frac{3}{6}n + 20 - \frac{3}{6}n = \frac{4}{6}n - \frac{3}{6}n \quad \text{Subtract } \frac{3}{6}n.
\]
\[
20 = \frac{1}{6}n \quad \text{Simplify.}
\]
\[
6(20) = 6\left(\frac{1}{6}n\right) \quad \text{Multiply by 6.}
\]
\[
120 = n \quad \text{Simplify.}
\]

70. The sum of one half of a number and 6 equals one third of the number.

**SOLUTION:**
\[
\frac{1}{2}n + 6 = \frac{1}{3}n \quad \text{Original.}
\]
\[
\frac{3}{6}n + 6 = \frac{2}{6}n \quad \text{Use LCD.}
\]
\[
\frac{3}{6}n - \frac{2}{6}n + 6 = \frac{2}{6}n - \frac{2}{6}n \quad \text{Subtract } \frac{2}{6}n.
\]
\[
\frac{1}{6}n + 6 = 0 \quad \text{Simplify.}
\]
\[
\frac{1}{6}n = -6 \quad \text{Add } -6.
\]
\[
6\left(\frac{1}{6}n\right) = 6(-6) \quad \text{Multiply by 6.}
\]
\[
n = -36 \quad \text{Simplify.}
\]

71. **SHOE** If \( \ell \) represents the length of a man’s foot in inches, the expression \( 2\ell - 12 \) can be used to estimate his shoe size. What is the approximate length of a man’s foot if he wears a size 8?

**SOLUTION:**
\[
8 = 2\ell - 12
\]
\[
8 + 12 = 2\ell - 12 + 12 \quad \text{Add 12 to each side.}
\]
\[
20 = 2\ell \quad \text{Simplify.}
\]
\[
\frac{20}{2} = \frac{2\ell}{2} \quad \text{Divide each side by 2.}
\]
\[
10 = \ell \quad \text{Simplify.}
\]

The approximate length of the man’s foot is 10 inches.
2-5 Solving Equations Involving Absolute Value

Write an equation for each problem. Then solve the equation.

72. Seven times a number equals –84. What is the number?

\[ \text{SOLUTION:} \]
\[ 7n = -84 \]
\[ n = -\frac{-84}{7} \quad \text{Divide each side by 7.} \]
\[ n = -12 \quad \text{Simplify.} \]

73. Two fifths of a number equals –24. Find the number.

\[ \text{SOLUTION:} \]
\[ \frac{2}{5}n = -24 \]
\[ \frac{5}{2} \left( \frac{2}{5}n \right) = \frac{5}{2}(-24) \quad \text{Multiply each side by} \frac{5}{2} \]
\[ n = -60 \quad \text{Simplify.} \]

74. Negative 117 is nine times a number. Find the number.

\[ \text{SOLUTION:} \]
\[ -117 = 9n \]
\[ \frac{1}{9}(-117) = \frac{1}{9}(9n) \quad \text{Multiply each side by} \frac{1}{9} \]
\[ -13 = n \quad \text{Simplify.} \]

75. Twelve is one fifth of a number. What is the number?

\[ \text{SOLUTION:} \]
\[ 12 = \frac{1}{5}n \]
\[ 5(12) = 5\left( \frac{1}{5}n \right) \quad \text{Multiply each side by 5} \]
\[ 60 = n \quad \text{Simplify.} \]